The optimal threshold for drug decriminalization: A game-theoretic model*

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Abstract

Despite growing interest in drug decriminalization, existing studies on decriminalization thresholds often rely on qualitative reasoning, resulting in unclear and often conflicting policy recommendations. This paper develops a dynamic game-theoretic model, grounded in the Beckerian economics of crime, to examine strategic interactions between the state and a representative drug dealer. In this framework, the state first sets the decriminalization threshold and penalty level, while the dealer later chooses the profit-maximizing drug quantity. The subgame perfect Nash equilibrium yields several key implications: the optimal threshold should always exceed average personal consumption, whilst penalties must remain harsh. In light of the theoretical result, the current methamphetamine threshold of 0.02 grams in Thailand should be increased by at least sixtyfold to approximately 1.2 grams, an amount on par with standards in Canada, the Czech Republic, and Portugal.

Keywords: Drug decriminalization policy, dynamic game with complete information, optimal penalty scheme

JEL classification codes: C7, K4

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1 Introduction

Recent decades have seen a significant global change in anti-drug policies from a seemingly failing war on drugs towards the opposite extreme of drug decriminalization. This decriminalization measure covers a wide range of leniency that spans from full legalization where drugs are allowed to be produced and consumed to partial decriminalization where the consumption, but not the production, of drug is either allowed or considered a mere administrative rather than a criminal offense. For a comprehensive summary of current drug policies around the world, see The Global Commission on Drug Policy (2021).

The development in these diverse forms of decriminalization policy has been followed in parallel with a plethora of related research that falls into two categories. The first category comprises the majority of research that focuses on the effects of the policy in a multitude of dimensions, which include, as summarized by Unlu et al. (2020), impacts on drug prevalence and the health of drug users, incidence of drug-related deaths and diseases, crime, and drug prices. The second category focuses on the often neglected question of what a good decriminalization policy should be, with an expansive corpus of literature centered around a key distinction between mere consumption versus distribution of drugs. At present, globally adopted practice of this distinction is often made by defining a threshold of the quantity of drug an individual may possess. A person found in possession of drugs would *ipso facto* be regarded as either a distributor or a mere user of drugs, depending on whether the quantity possessed exceeds this threshold. Whilst such practice is, *faute de mieux*, widely accepted due to its relative ease of execution and its potential use as a buffer against unfair discretionary judgment of law enforcement officers, how the practice should actually be implemented remains unclear and controversial. As Harris (2011) observes, the determinants of decriminalization thresholds vary significantly across countries. Germany and Estonia define thresholds qualitatively (e.g., “small” vs. “large” amounts), Norway uses standard doses, and Portugal bases its criteria on a number of days’ supply. Despite these differences, a shared legal philosophy prevails: drug users and dealers should be treated distinctly, and thresholds should reflect empirical patterns of personal consumption.

An instructive example of how the absence of a clear and rational guideline for determining the appropriate threshold of drug possession can lead to policy uncertainty and inconsistency can be found in Thailand. Following the seemingly unsuccessful war on drugs waged under the premiership of Thaksin Shinawatra in 2003, Thailand enacted a dramatic shift in its anti-drug policy, introducing a decriminalization measure whereby an individual in possession of no more than five methamphetamine pills would be classified not as a criminal offender but as a person afflicted by mental illness, eligible for rehabilitation services in state hospitals. But in response to the recent administration's resolve to impose stricter measures amid rising concerns over escalating drug-related problems, the Office of Narcotics Control Board (ONCB) has reduced this threshold to a single pill in 2024. This approach to determining the optimal limit nonetheless appears to be largely algorithmic in nature and devoid of any discernible theoretical foundation; it first considered a reduction of the threshold down from five to three pills, then stated that this revision would be contingent upon an assessment of the drug situation after the policy implementation — if the number of users was found to exceed that of dealers, the legal limit would be further reduced (The Editorial Board, 2024).

The lack of guidance for an effective decriminalization policy as illustrated by the above example may, in part, be attributed to the scarcity of literature on the optimal threshold, and to the fact that the meager research that does exist relies mostly upon qualitative analysis and argumentative reasoning. Manderson (2022), for example, recognizes the subjective nature of this threshold and proposes that the threshold be updated on a regular basis vis-à-vis the personal consumption of different types of drugs and consumption behaviors yet emphasizes that it should be *independent* of the relative social harm of each type of drug. Another study by McAdam et al. (2023) attempts to provide a guideline for constructing the threshold using a multi-criteria policy analysis framework, given the limitation, as the authors point out, that lies in the subjectivity of the approach. Harris (2011) argues that the use of a single *a priori* threshold may not account for the heterogeneity of drug use and thus amounts to an "immeasurable cost of injustice".

This paper seeks to contribute to the literature on the decriminalization threshold by constructing a game-theoretic model that disentangles all the aforementioned issues, i.e. the nexus between the optimal threshold and personal consumption, the heterogeneity in drug use behaviors and the plausible immeasurable cost of injustice, and the interdependency of a threshold and the social cost of drugs — all in a unified and systematic framework. The paper also simultaneously explores the use of an equally important yet relatively understudied decriminalization measure: the severity of punishment for drug offenses together with the decriminalization threshold to constitute an optimal penalty scheme that maximizes the social benefit. To the best of my knowledge, no quantitative study has yet been undertaken to determine the optimal threshold for drug decriminalization. Nevertheless, the model developed in this article is grounded in the well-established principles of the economics of crime, drawing upon a body of literature that has long shaped discourse in this domain.

Since Becker's (1968) seminal formulation of crime as a rational choice problem, the literature on the optimal design of enforcement policies has expanded considerably. Polinsky and Shavell (2000) survey this literature and categorize key policy questions, including how much to invest in apprehension, whether liability should be strict or fault-based, whether sanctions should be monetary or custodial, and how penalties should vary with harm, intent, or reporting behavior, and how optimal sanctions differ for corporate versus individual offenders. A specific strand of this literature employs game-theoretic models to analyze strategic interaction between the state and offenders. Avenhaus et al. (2002) model inspection as a sequential game where the inspector, subject to resource constraints, acts as a leader. Andreoni et al. (1998) emphasize repeated interaction in tax compliance, where enforcement credibility affects compliance behavior. In the context of corruption, Mookherjee and Png (1995) show that raising penalties for corruption can perversely increase pollution, while Iyavarakul (2009) finds a trade-off between bribe prevalence and bribe size under harsher anti-corruption measures. Miceli (1990, 1991) incorporates both deterrence and fairness in the social welfare function, showing how the optimal penalty must balance crime prevention and just treatment of the accused.

Of particular relevance to this paper is a subset of the economics of crime literature that theoretically models illicit drug markets. Behrens et al. (2000) apply optimal control to drug prevention and treatment policies, showing that prevention is most effective early in an epidemic, treatment is more effective later, and delays in intervention substantially raise social costs. Tragler et al. (2001) also use optimal control and find that if control begins early, intensive use of both enforcement and treatment can suppress the epidemic; otherwise, the optimal policy is to moderate its growth. Poret and Téjédo (2006) model traffickers' detection risk as increasing with market share and show that stricter enforcement may induce market entry, even as it reduces individual supply. As a result, total drug supply and criminal profits remain largely unaffected by enforcement intensity.

In light of the foregoing literature, this paper makes three primary contributions. First, this paper applies a dynamic game-theoretic model—grounded in the Beckerian tradition—to the issue of drug decriminalization. While similar game-theoretic models have been developed within the economics of crime literature, this is, to the author's best knowledge, the first application to the modeling of decriminalization policy. Second, although the literature on drug decriminalization outside economics is substantial, it remains largely qualitative. This paper contributes a formal, quantitative framework that provides structured insights into a policy area previously dominated by normative and descriptive analyses. Third, the model generates a concrete recommendation for Thailand's ongoing debate on the appropriate threshold for drug decriminalization. While motivated by the Thai context, the analysis is generalizable and may inform policy design in other countries pursuing or reconsidering decriminalization.

Briefly foreshadowing the findings, the model suggests that the optimal threshold of drug possession should always be set higher than the average consumption of drugs *and* should be increasing in the social costs of drug abuse, a result that both complements and contrasts Manderson (2022). Also, whilst all countries should allow for some degree of decriminalization, it is optimal to set the penalty level for drug offenses beyond that which society deems fit for the crime.

2 The model

Consider a one-shot two-stage dynamic game between the state and a representative drug dealer. The state sets a penalty scheme for an individual found to possess an amount of drugs beyond a specified threshold in the first stage. Upon observing this, in the second stage the dealer chooses a quantity of drug produced to maximize profit. The state commits to its strategy. Both players have complete information regarding the payoffs for all players.

The model developed in the paper emphasizes the interaction between the state and a representative drug dealer. Under the assumption that dealers have homogeneous preferences, this representative-agent approach simplifies the analysis by allowing us to focus on the central questions of how decriminalization policy influences dealer behavior and how such policy ought to be designed, without necessitating detailed consideration of strategic interactions among sellers. This model is well-suited to market structures in which illicit drug markets are either perfectly competitive or monopolistically competitive, and where interactions among sellers are negligible or minimal. The choice to frame the analysis as a one-shot game reflects the fact that drug policies in most countries are rarely revised within a single administration. In practice, the state's decision to repeal its policy in response to dealer behavior is often subject to a complex legislative process and therefore tends to be less immediate than dealers' adaptation to legal changes. In this spirit, the paper aims to develop a framework that captures the essence of this strategic interaction in a one-shot rather than a repeated-game setting.

The profit of the drug dealer is defined by

$$\Pi = px - c(x) - \lambda k(x; \bar{x}, \delta), \tag{1}$$

where x is the quantity of drug produced, p is a constant per unit price of drug, $c(x)$ is the cost function of drug production, such that $c_x(x) > 0$ and $c_{xx}(x) \geq 0$. The term $\lambda k(x; \bar{x}, \delta)$ is an additional expected cost that reflects the illegality of the drug, where $0 < \lambda < 1$ is the probability that the dealer is arrested and convicted. Indeed, λ represents the probability that a drug dealer is ultimately subjected to punishment. This term is, in essence, the product of a sequence of conditional probabilities at each stage of the justice process: the probability of arrest given that the quantity sold exceeds the legal threshold; the probability of prosecution given arrest; the probability of conviction given prosecution; and the probability of punishment given conviction. Collectively, this concept is succinctly described as the probability of arrest and conviction. Finally, $k(x; \bar{x}, \delta)$ is a penalty function that imposes an additional cost on an individual possessing drugs over the threshold $\bar{x}$, with a severity parameter δ, such that (1) $k(x; \bar{x}, \delta)$ is zero when $x \leq \bar{x}$ and positive otherwise; (2) $k(x; \bar{x}, \delta)$ is differentiable at any x; (3) $k_x(x; \bar{x}, \delta) > 0$ and $k_{xx}(x; \bar{x}, \delta) \geq 0$ for $x > \bar{x}$; and (4) $k_\delta(x; \bar{x}, \delta) > 0$ for $x > \bar{x}$.

The optimal quantity satisfies the first order condition

$$\Pi_x = p - c_x(x) - \lambda k_x(x; \bar{x}, \delta) \ = 0, \tag{2}$$

which can be rearranged to get

$$p - c_x(x) = \lambda k_x(x; \bar{x}, \delta),$$

with a solution of the form $x^* = x(p, \lambda, \bar{x}, \delta)$. Results from a comparative static analysis of $x(p, \lambda, \bar{x}, \delta)$ are summarized as follows.

Proposition 1 If both $c_{xx}(x)$ and $k_{xx}(x; \bar{x}, \delta)$ are non-decreasing and at least one of them is strictly increasing, then $x(p, \lambda, \bar{x}, \delta)$ is (1) strictly increasing in p for all x, (2) strictly increasing in $\bar{x}$ for $x > \bar{x}$ and $k_{x\bar{x}}(x; \bar{x}, \delta) < 0$, (3) strictly decreasing in λ for $x > \bar{x}$, and (4) strictly decreasing in δ for $x > \bar{x}$ and $k_{x\delta}(x; \bar{x}, \delta) > 0$. ■

Proof Totally differentiating the first order condition with respect to $p, \lambda, \bar{x}, \delta$

yields

$$\begin{aligned}&\Pi_{xx}dx + \Pi_{xp}dp + \Pi_{x\lambda}d\lambda + \Pi_{x\bar{x}}d\bar{x} + \Pi_{x\delta}d\delta \\ &= -\left(c_{xx}(x) + \lambda k_{xx}(x;\bar{x},\delta)\right)dx + dp - k_x(x;\bar{x},\delta)d\lambda \\ &\quad -\lambda k_{x\bar{x}}(x;\bar{x},\delta)d\bar{x} - \lambda k_{x\delta}(x;\bar{x},\delta)d\delta = 0.\end{aligned}$$

Since the denominator of the following partial derivatives is positive when both $c_{xx}(x)$ and $k_{xx}(x;\bar{x},\delta)$ are non-decreasing and at least one of them is strictly increasing, hence

$$x_p = \frac{1}{c_{xx}(x) + \lambda k_{xx}(x;\bar{x},\delta)} > 0.$$

Similarly,

$$x_{\bar{x}} = -\frac{\lambda k_{x\bar{x}}(x;\bar{x},\delta)}{c_{xx}(x) + \lambda k_{xx}(x;\bar{x},\delta)} > 0,$$

if $k_{x\bar{x}}(x;\bar{x},\delta) < 0$. Also

$$x_\lambda = -\frac{k_x(x;\bar{x},\delta)}{c_{xx}(x) + \lambda k_{xx}(x;\bar{x},\delta)} < 0,$$

and

$$x_\delta = -\frac{\lambda k_{x\delta}(x;\bar{x},\delta)}{c_{xx}(x) + \lambda k_{xx}(x;\bar{x},\delta)} < 0,$$

if $k_{x\delta}(x;\bar{x},\delta) > 0$. Note that the above results on $x_{\bar{x}}, x_\lambda, x_\delta$ only hold for $x > \bar{x}$, since $k(x;\bar{x},\delta) = 0$ for $x < \bar{x}$ by definition. □

Intuitively, the effect of the penalty-related parameters only affects the dealer whose optimal quantity exceeds the legal threshold $\bar{x}$. Under the assumptions that the marginal cost of punishment $k_x(x;\bar{x},\delta)$ is strictly decreasing in the threshold parameter $\bar{x}$, and strictly increasing in the severity parameter δ, the quantity of drugs produced will respond to the penalty parameters in a rational way, such that it increases in the legal threshold, but decreases in punishment severity.

In the first stage, the state chooses the threshold $\bar{x}$ and the severity parameter δ to minimize the social cost function, which is similar in spirit to Miceli (1990) but modified to suit our context and assumed to be comprised of three components. The first component is contingent upon the rampancy of drug usage, which can be deterred by either a lowering threshold or an increasing punishment severity. The second component captures the retribution effect of the penalty in terms of the discrepancy between the imposed punishment level δ and the just level of severity $\bar{\delta}$ that best fits the crime. If $\delta < \bar{\delta}$, the law is too lenient, whilst if $\delta > \bar{\delta}$, the law is too harsh.[1] The last component is the conviction error, namely, a case in which a drug user who possesses the drugs in excess of $\bar{x}$ is falsely convicted and punished on a drug dealing charge. The social cost function is assumed to be additively separable in all these three components of the form

$$\Gamma = \alpha u(x(p, \lambda, \bar{x}, \delta)) + \beta v(||\delta - \bar{\delta}||) + \gamma w\left(\lambda\left(1 - F_Y(\bar{x})\right)\right), \tag{3}$$

where $\alpha, \beta, \gamma > 0$ are the weights corresponding to each component of the social cost; $u(\cdot), v(\cdot)$, and $w(\cdot)$ are strictly increasing functions; $x(p, \lambda, \bar{x}, \delta)$ is the profit maximizing quantity of the dealer in the second stage; $||\delta - \bar{\delta}||$ is the Euclidean distance between δ and $\bar{\delta}$; and $F_Y(y)$ is the cumulative probability distribution function of the *exogenous* quantity of drug y that a drug user possesses.[2]

Under the assumption that the operation cost of $\bar{x}$ and δ is negligible, the state solves an unconstrained optimization problem with corresponding

[1] The concept of a just level of severity ($\bar{\delta}$) is grounded in retributive legal philosophy, which holds that punishment must be strictly proportionate to the harm caused. The maxim from Hammurabi's code—an eye for an eye—illustrates this idea: if an offender destroys an eye, justice requires forfeiture of one eye, no more and no less. This stands in contrast to the optimal level of severity (δ^*), which is determined by broader considerations of social welfare. Under an optimal framework as posited by the minimization of the social cost function in equation (3), the appropriate punishment takes into account not only the retributive purpose of sentencing but also factors such as the deterrent effect of penalties and the need to avoid punishing the innocent.

[2] Assuming that drug consumption follows a cumulative distribution function F_Y, it follows that $(1 - F_Y(\bar{x}))$ is the probability that a drug user possesses an amount exceeding the legal threshold $\bar{x}$, and will therefore be misclassified and prosecuted as a drug dealer. Given that λ denotes the probability of conviction, the expected probability that a drug user is falsely punished is $\lambda(1 - F_Y(\bar{x}))$. This term forms the final component of the social cost, reflecting the societal concern over drug users being wrongly punished as dealers.

first order conditions

$$\Gamma_{\bar{x}} = \alpha u_x x_{\bar{x}} - \gamma \lambda w_z f_Y(\bar{x}) = 0, \tag{4}$$

$$\Gamma_{\delta} = \alpha u_x x_{\delta} + \beta v_d \text{sgn}(\delta - \bar{\delta}) = 0, \tag{5}$$

where $z = \lambda(1 - F_Y(\bar{x}))$, $d = ||\delta - \bar{\delta}||$, and $\text{sgn}(\cdot)$ is the sign function. The first equation states that any change in the threshold carries two contrasting effects on the social cost. A lower threshold, for example, will decrease the social cost through an increasing deterrent effect, but at the same time it also raises the social cost as it increases the probability of falsely convicting a drug user. The state should therefore seek to find a threshold that balances the two effects and, provided that the solution is interior, i.e. $\bar{x}^* > 0$, allows for some degree of decriminalization.

An increase in the punishment severity that is initially below the just level, i.e. $\delta < \bar{\delta}$ will *always* decrease the social cost as such increase will produce a higher deterrent effect and bring the punishment severity closer to the just level. Indeed, it is straightforward to verify that $\Gamma_\delta < 0$ when $\delta < \bar{\delta}$ so that the state can continue to decrease the social cost by increasing δ. When $\delta > \bar{\delta}$, an increase in the punishment severity will generate contrasting effects from an increasing deterrent effect of a harsher punishment as well as an increasing burden on the social cost as the punishment severity moves further away from the just level. Figure 1 illustrates the optimality condition for the punishment severity; the optimal penalty severity δ^* is where the marginal deterrent effect line $\alpha u_x x_\delta$ intersects the marginal retribution effect line $-\beta v_d \text{sgn}(\delta - \bar{\delta})$ from above. An increase in the penalty severity that is below the just level $\bar{\delta}$ always has a positive effect on the social welfare in terms of the marginal reduction in the social cost $-\Gamma_\delta$ since such increase will both increase the deterrent effect and bring the punishment closer to the just level. An increase in the penalty severity beyond the just level, however, produces a trade-off between an increase in social welfare from a stronger deterring effect and a decrease in welfare from a weaker retribution effect. The optimal penalty severity must therefore always exceed the just level.

The following proposition summarizes this result.

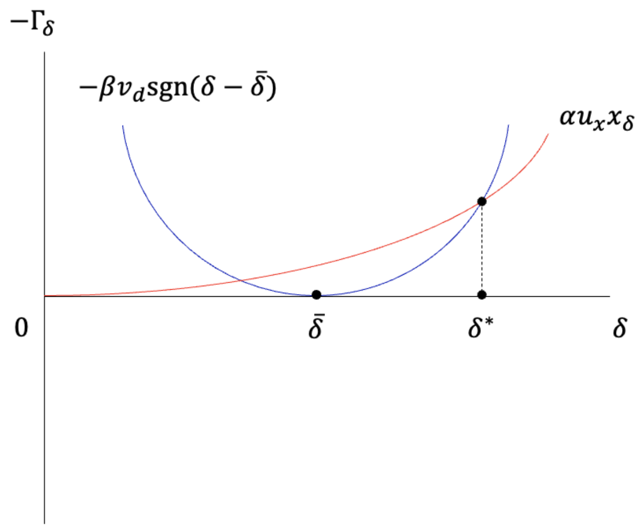

Figure 1: The optimal penalty severity δ^* is where the marginal deterrent effect line $\alpha u_x x_\delta$ intersects the marginal retribution effect line $-\beta v_d \text{sgn}(\delta - \bar{\delta})$ from above. For any $\delta < \bar{\delta}$, an increase in δ only carries marginal benefit from both an increasing deterrent effect and the punishment level that is brought closer to the just level $\bar{\delta}$. For any $\delta \geq \bar{\delta}$, an increase in δ induces a marginal benefit, from an increasing deterrent effect, and the marginal cost, as δ is pushed further away from the just level $\bar{\delta}$. The optimal penalty severity must therefore always exceed the just level.

Proposition 2 The optimal penalty level always exceeds the just level, i.e. $\delta^* > \bar{\delta}$. ■

The solution to the above system of typically non-linear normal equations with two equations and two unknowns provides the socially optimal penalty scheme $\bar{x}(p, \lambda, \alpha, \beta, \gamma)$ and $\delta(p, \lambda, \alpha, \beta, \gamma)$.

3 A parametric example

To ensure that an analytical solution of x^* exists, suppose the cost function is linear in quantity, i.e. $c(x) = cx$, where $c > 0$ is a constant, and the penalty function $k(x; \bar{x}, \delta)$ is a piecewise polynomial function of the form

$$k(x; \bar{x}, \delta) = \begin{cases} (x - \bar{x})^{\delta} & \text{if } x > \bar{x} \\ 0 & \text{otherwise,} \end{cases} \tag{6}$$

where $1 < \delta \leq \frac{p-c}{\lambda}$ is the severity parameter chosen by the legislature. Since $\lim_{x \to \bar{x}^{(-)}} k_x(x; \bar{x}, \delta) = 0$ and $\lim_{x \to \bar{x}^{(+)}} k_x(x; \bar{x}, \delta) = \delta(x - \bar{x})^{\delta - 1} = 0$, $k(x; \bar{x})$ is continuous and differentiable at any x. It is also straightforward to verify that for $x > \bar{x}$, $k_x(x; x, \delta) > 0$, $k_{x\bar{x}}(x; \bar{x}, \delta) < 0$, and $k_{x\delta}(x; \bar{x}, \delta) > 0$ and thus the results in Proposition 1 hold.

Upon substitution, the profit function of the dealer becomes

$$\Pi = (p - c)x - \lambda(x - \bar{x})^{\delta} \mathcal{I}(x > \bar{x}), \tag{7}$$

where $\mathcal{I}(\cdot)$ is the indicator function equal to 1 if the argument is true, and 0 otherwise, and p is a constant per unit price of the drug.

Proposition 3 The quantity of drug that maximizes the profit function of the dealer as defined in (1) is

$$x^* = \begin{cases} 0 & \text{if } p \leq c \\ \bar{x} + \tilde{p}^{\frac{1}{\delta - 1}} & \text{otherwise,} \end{cases} \tag{8}$$

where $\tilde{p} = \frac{p-c}{\lambda\delta} > 0$. ■

Proof Absent the expected cost of penalty, the solution to a profit maximization problem with a linear cost function is a corner solution, where $x^* = 0$ if $p \leq c$. If $p > c$, $x < \bar{x}$ cannot maximize profit since the profit function is strictly increasing in x, for $0 \leq x < \bar{x}$. For $x^* \geq \bar{x}$, the profit-maximizing quantity satisfies the first order condition

$$p - c = \lambda\delta(x^* - \bar{x})^{\delta-1},$$

or that

$$x^* = \bar{x} + \left(\frac{p-c}{\lambda\delta}\right)^{\frac{1}{\delta-1}} = \bar{x} + \tilde{p}^{\frac{1}{\delta-1}}. \quad \square$$

The term $\tilde{p} = \frac{p-c}{\lambda\delta}$ can be interpreted as the *effective* profit margin of the drug dealer. A comparative static analysis of x^* produces intuitive results according to Proposition 1 — that x^* is increasing in drug price and the threshold level but decreasing in the production cost, the probability of detection, and the punishment severity.

In the first stage, suppose that $u(x) = x$, $v(||\delta - \bar{\delta}||) = (\delta - \bar{\delta})^2$, and $w\left(\lambda\left(1 - F_Y(\bar{x})\right)\right) = \lambda(1 - \Phi(\bar{x}))$, where $\Phi(\cdot)$ is the cumulative distribution function of a normal distribution with expectation μ and variance σ^2. The state therefore chooses $\bar{x}$ and δ in the first stage to minimize

$$\Gamma = \alpha\left[\bar{x} + \tilde{p}^{\frac{1}{\delta-1}}\right] + \beta(\delta - \bar{\delta})^2 + \gamma\lambda(1 - \Phi(\bar{x})), \tag{9}$$

with corresponding first order conditions

$$\Gamma_{\bar{x}} = \alpha - \gamma\lambda\phi(\bar{x}) = 0 \text{ or that } \phi(\bar{x}) = \frac{\alpha}{\gamma\lambda}, \tag{10}$$

$$\Gamma_{\delta} = -\frac{\alpha}{\delta-1}\tilde{p}^{\frac{1}{\delta-1}}\left[\frac{1}{\delta-1}\ln\tilde{p} + \frac{1}{\delta}\right] + 2\beta(\delta - \bar{\delta}) = 0, \tag{11}$$

where $\phi(\cdot)$ is the corresponding probability density function of $\Phi(\cdot)$. These first order conditions constitute a system of separable non-linear equations such that $\bar{x}$ and δ can be respectively identified from the first and the second equation alone. For example, the first equation, together with the sufficient condition for minimization that the second derivative must be positive, i.e.

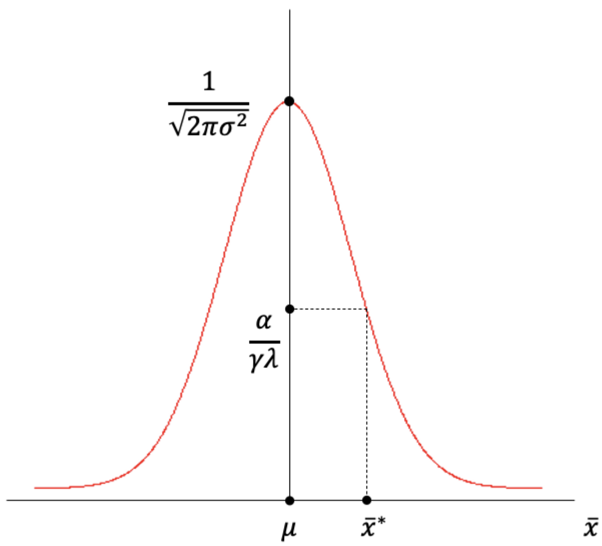

Figure 2: (a) The optimal legal threshold $\bar{x}^*$ must satisfy the first order condition $\phi(\bar{x}) = \frac{\alpha}{\gamma\lambda}$ and the second order condition $\phi'(\cdot) < 0$. Hence, $\bar{x}^* > \mu$. Note that since $\phi(\cdot)$ reaches the maximum at $\bar{x} = \mu$, with $\phi(\mu) = \frac{1}{\sqrt{2\pi\sigma^2}}$, the solution exists if and only if $\sqrt{2\pi\sigma^2} < \frac{\gamma\lambda}{\alpha}$.

$\phi'(\cdot) < 0$ or that $\bar{x} > \mu$, yields a unique value for the optimal threshold, $\bar{x}^*$, which always exceeds the average quantity of drug consumption as shown in the following proposition.

Proposition 4 The optimal threshold of drug allowance is

$$\bar{x}^* = \mu + \left[-2\sigma^2 \ln\left(\frac{\alpha\sqrt{2\pi\sigma^2}}{\gamma\lambda}\right)\right]^{\frac{1}{2}}, \tag{12}$$

where $\bar{x}^* > \mu$. ■

Proof Given that the optimal legal threshold $\bar{x}^*$ must satisfy the first order condition $\phi(\bar{x}) = \frac{\alpha}{\gamma\lambda}$, and $\phi(\cdot)$ reaches the maximum at $\bar{x} = \mu$, with $\phi(\mu) = \frac{1}{\sqrt{2\pi\sigma^2}}$, the solution exists if and only if $\sqrt{2\pi\sigma^2} < \frac{\gamma\lambda}{\alpha}$. This requirement also guarantees that the term in the bracket on the right hand side of $\bar{x}^*$ in Proposition 4 is positive. Hence, $\bar{x}^* > \mu$ or that the optimal threshold or drug allowance always exceeds the average quantity of drug consumption. □

It is straightforward to verify, as is evident from the graphical illustration of the first order condition in Figure 2, that $\bar{x}^*$ is increasing in μ, γ, and λ, but decreasing in α. The interpretation of this comparative static result is intuitive. An increase in the expected quantity of drug possessed among users will accordingly raise the optimal legal threshold, as the previous threshold will incur too high a cost of mistakenly penalizing a user as a dealer. Also, the optimal legal threshold is lower in a society that places more emphasis on deterring drug use, and higher in a society that has a greater concern on falsely convicting an "innocent" person. Finally, a decrease in the probability of conviction requires that the optimal legal threshold be lower to bring about the same deterrent effect.

The effect of σ^2, the variance of the quantity of drugs possessed among the users, on $\bar{x}^*$ is nevertheless unclear. Since

$$\bar{x}^*_{\sigma^2} = \frac{1}{2}\left[-2\sigma^2 \ln\left(\frac{\alpha\sqrt{2\pi\sigma^2}}{\gamma\lambda}\right)\right]^{-\frac{1}{2}}\left[-2\ln\left(\frac{\alpha\sqrt{2\pi\sigma^2}}{\gamma\lambda}\right) - 1\right], \tag{13}$$

and $\sqrt{2\pi\sigma^2} < \frac{\gamma\lambda}{\alpha}$, $\bar{x}^*_{\sigma^2} > 0$ if and only if the term in the second bracket is

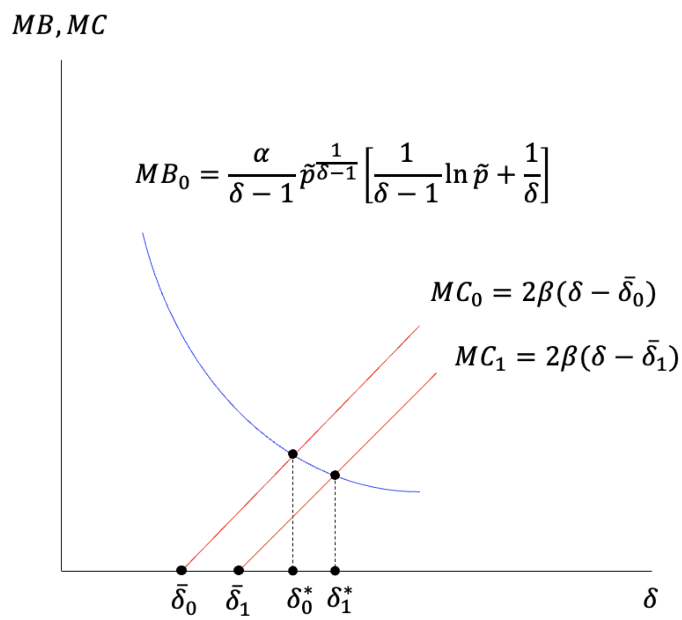

Figure 3: The first order condition determines the optimal penalty level $\delta^* > \bar{\delta}$ at the point where the marginal social benefit from the deterrent effect of an increase in the penalty level, MB, intersects the marginal social cost from the retribution effect when the imposed penalty level shifts further away from the just level, MC. A comparative static analysis of the effect of the change in the model parameters on δ^* is clear from the diagram. For example, an increase in $\bar{\delta}$ from $\bar{\delta}_0$ to $\bar{\delta}_1$ will shift the marginal social cost line rightward from MC_0 to MC_1, thus resulting in an increase in the optimal penalty level from δ_0^* to δ_1^*.

positive. Hence, the optimal legal threshold is increasing in the variance of drug consumption when $\frac{\alpha}{\gamma\lambda}$ is relatively small. Intuitively, when the pattern of drug consumption becomes more dispersed—that is, when σ^2 increases—the state is more likely to falsely convict a drug user as a dealer, thereby raising the social cost through its final component. Accordingly, the optimal threshold should be set higher as the dispersion of consumption increases when society places greater weight on the cost of false convictions relative to the cost of the prevalence of drug; that is, when $\frac{\alpha}{\gamma\lambda}$ is relatively small.

Although it is evident from the second first order condition that an analytical solution for δ^* does not exist, a geometric representation of this equation provides an appealing economic interpretation of this result. As shown in Figure 3, the marginal social cost from the retribution effect, $2\beta(\delta-\bar{\delta})$, is a positively sloped linear line with a horizontal intercept at $\bar{\delta}$. Given the *a priori* assumption $1 < \delta \leq \frac{p-c}{\lambda}$, the marginal social benefit from the deterrent effect, $\frac{\alpha}{\delta-1}\tilde{p}^{\frac{1}{\delta-1}}\left[\frac{1}{\delta-1}\ln\tilde{p}+\frac{1}{\delta}\right]$, is positive and can be shown to be nonlinearly decreasing in δ. The optimal penalty level δ^*, which is determined by the intersection of these two curves, must therefore lie to the right of $\bar{\delta}$, a result consistent with Proposition 2. Comparative static analysis results of the change in the model parameters on δ^* can also be inferred directly from the diagram. In particular, δ^* is increasing in α, p, and $\bar{\delta}$, and decreasing in β, c, and λ. For example, an increase in $\bar{\delta}$ from $\bar{\delta}_0$ to $\bar{\delta}_1$ will shift the marginal social cost line leftward, resulting in an increase in the optimal penalty level from δ_0^* to δ_1^*.

This section is concluded, in passing, with an examination on the robustness of the result stated in Proposition 4. While the result in Proposition 4— that the optimal threshold must exceed the average consumption level of the drug (i.e., $\bar{x}^* > \mu$) — still relies on the algebraic convenience of an analytical solution of $\bar{x}$ obtained under assumptions of a linear production cost and a polynomial penalty function, it is straightforward to show that the result also holds for any consumption distribution that is symmetric and unimodal around its expectation. This encompasses a wider range of distributions beyond the normal distribution, including the t-distribution, the Laplace distribution, and certain types of beta distributions. To see this, suppose that the consumption of drug Y follows a unimodal distribution that is symmetric around its expectation μ, with a cumulative distribution function $F_Y(\cdot)$ and a probability density function $f_Y(\cdot)$. The social cost function in (9) thus becomes

$$\Gamma = \alpha \left[\bar{x} + \tilde{p}^{\frac{1}{\delta-1}}\right] + \beta(\delta - \bar{\delta})^2 + \gamma\lambda(1 - F_Y(\bar{x})), \tag{14}$$

with corresponding first-order conditions

$$\Gamma_{\bar{x}} = \alpha - \gamma\lambda f_Y(\bar{x}) = 0, \tag{15}$$

$$\Gamma_{\delta} = -\frac{\alpha}{\delta - 1}\tilde{p}^{\frac{1}{\delta-1}} \left[\frac{1}{\delta - 1} \ln \tilde{p} + \frac{1}{\delta}\right] + 2\beta(\delta - \bar{\delta}) = 0. \tag{16}$$

From (15),

$$\bar{x}^* = f_Y^{-1}\left(\frac{\alpha}{\gamma\lambda}\right). \tag{17}$$

The second-order condition for cost minimization requires that $f'_Y(\bar{x}^*) < 0$. Together with the properties of a distribution function that is symmetric arounds its expectation, i.e. $f'(y) \geq 0$ for $y < \mu$ and $f'(y) \leq 0$ for $y > \mu$, there exists a unique solution to the social cost minimization problem in (14) such that $\bar{x}^* > \mu$. Indeed, the result in equation (17), together with the second-order condition, can be generalized to imply that the optimal threshold must exceed the *mode* of the consumption distribution that is unimodal but not necessarily symmetric around the mean.

4 Discussion

Despite a growing body of literature on drug decriminalization, existing studies rely mostly on a qualitative approach and argumentative reasoning. Policy recommendations regarding this issue thus remain contradictory. This paper proposes a game-theoretic approach in modeling the policy and finds that the subgame perfect Nash equilibrium of the model can reinforce and reconcile several policy implications suggested by the previous literature. Notably, the optimal threshold should always be set relative to the average personal consumption, a result consistent with recommendations by Manderson (2022) and McAdam et al. (2023). Our model further stipulates the exact nature of this relationship and argues that the threshold must always be greater than the average personal consumption, by a magnitude that is determined by factors including the social cost of illegal drug usage. The optimal penalty on individuals who possess drugs over the threshold should, however, be kept harsh, to the level beyond which society deems fit the crime. Whilst this practice may exact the "immeasurable cost of injustice" as argued by Harris (2011), it is worth emphasizing that such concern is embedded in, rather than ignored by, the model, as an increase in the magnitude of such costs will necessitate a corresponding adjustment in the threshold. For instance, a society that considers the social cost of erroneously accusing an innocent individual to be infinitely high would consequently choose a threshold that approaches infinity. This threshold effectively corresponds to the complete legalization of drugs.

In retrospect, the principal finding of this article, that the optimal threshold must necessarily exceed the average level of consumption, serves to underscore the inherent irrationality of the ONCB's decision to reduce the threshold to a mere one pill, or approximately 0.02 grams. Given that the average quantity purchased for consumption in Thailand reached 59 pills per week in 2014 (Sukharomana, et al., 2014), the model implies that a far higher threshold, at least 1.2 grams, is warranted. Indeed, this proposed threshold remains significantly lower than the standards adopted by several other nations, such as Canada (2.5 grams), the Czech Republic (2 grams), and Portugal (2 grams) (Eastwood et al., 2016). While such a substantial increase in the threshold would no doubt create objections, particularly among social conservative, it is imperative to recognize that a range of policy alternatives is available for addressing the prevalence of drug use. Among these, and what may appear paradoxical, even the re-criminalization of drug consumption itself remains a conceivable, though extreme, measure should all other avenues prove ineffective. This, at the very least, would preserve the sanctity of the law by penalizing illegal drug usage as such. Thus it would seem to be in the interest of justice to hold drug users accountable for the actual offense they commit rather than to subject them to punishment on a spurious pretext—such as drug smuggling—which bears no relation to their actions.

Suggestions for future work include the examination of the robustness of these conclusions on different model assumptions. Some assumptions, i.e. that the social cost function is increasing and additively separable in the quantity of drug supplied, the Euclidean distance between the imposed and the just penalty severity, and the probability of falsely convicting a drug user as a dealer, are for algebraic convenience; another social cost function that is increasing but not necessarily additively separable in these components is expected to render results in the same reduction. A non-trivial extension of the model is to endogenize the consumption decision of drug users by modeling a dynamic game with three players, in which the state first selects the possession threshold and the penalty severity to minimize social cost, and the dealer and the user simultaneously select the amount of drug produced and consumed in the second stage. In this regard, it is important to note that while a fully specified economic model would typically analyze both sides of the market, the demand for illicit drugs is a complex matter that extends beyond the rationality of the *homo economicus* and often requires insights from multidisciplinary perspectives, including behavioral science and public health. The extent to which drug consumption responds to changes in policy variables, such as legal thresholds, remains a subject of considerable debate given that drug users face inherent constraints: consumption cannot be too high without risking overdose, nor too low due to the compulsive nature of addiction. See Becker and Murphy (1988), Gruber and Köszegi (2001), and Frederick et al. (2002) for examples of the literature on this topic.

Another interesting extension of the model is to endogenize drug price as one might expect it to be dependent on the production cost and the legal environment. The conclusion that the optimal threshold must always exceed the average drug consumption also relies fundamentally on the assumption that drug consumption is normally distributed. Whilst it has been shown that the same conclusion can be reached for any symmetric and unimodal probability distribution of drug consumption, other models that encompass different types of distribution certainly merit further research.

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