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Abstract

This paper aims to investigate the effects of macroeconomic factors to real estate bubble in Thailand. We develop the real estate bubble index by the principle component analysis method suggested by Union Bank of Switzerland. Five indicators, which represent demand growth, price pressure, financial sector, production, and supply growth, are selected to create the real estate bubble index of Thailand. Evidently, the index can explain the bubble dynamic in the Thailand's real estate market during 1993-2021 well. The investment, consumption and minimum loan rate are the important macroeconomic factors affecting the real estate bubble in Thailand. The credit to GDP ratio has insignificant effect to the movement of bubble. The total GDP has limited impacts on the RBI. Lastly, both conventional monetary policy (repurchase rate) and macro-prudential (loan-to-value ratio) tools have limited impacts on controlling bubbles.

Keyword: Real Estate bubble, Macroeconomic Factors, Bayesian Vector AutoRegression, Monetary Policy

JEL Classification: E44, E58, G12, L85

1. Introduction: Why are We Afraid of Real Estate Bubbles?

For the past two decades, world economy continuously experienced many economic crises associated with crashes of bubbles. Jorda, Schularick, and Taylor (2015) has rigorously shown the empirical evidence on the variety of bubbles and has highlighted that the one accompanied by the credit boom is the most dangerous. Miao, Wang, and Xu (2015) have shown that the crash of bubble leads to the recession with high and persistent unemployment. Japan's Bubble Economy in 1986-1991, East Asia Financial Crisis in 1994-1997, Dot-com collapse in 1997-2000, Subprime Crisis in 2003-2008, and Eurozone Crisis in 2009 until now are evident examples. Each of the crisis caused large-scale damages to its own economy and also contagiously affected its economic partners.

In most crises, economists find that the key common element that causes economic crises tends to be the fluctuation in asset price resulted from speculation. Speculation pushes the asset price and hence the entire economy on the boom. The boom continues until the market sentiment suddenly changes due to the recognition of artificial demand for asset. Consequently, there is a sudden collapse in the asset market which is followed by the fall of banking system. This boom-bust implies that there exist bubbles in the asset price.

Among the bubble in various asset classes, the real estate bubbles had the significant roles in the macroeconomy and financial sectors. The Tom Yum Kung crisis in Thailand and East Asia during 1997 to 1998 was characterized with booming in the real estate sectors. Similarly, the Global Financial Crisis started by the bubble burst in the US real estate price.

Consequently, this paper has two objectives. First, we aim to develop the indicator for real estate bubbles in Thailand. For this purpose, we adopt and improve the existing method of Union Bank of Switzerland (UBS) to develop a good bubble indicator that is consistent with the theory of rational bubbles and can detect bubbles effectively in the Thai real estate market. Second, we explore the policy issue by examining the roles of macroeconomic, financial markets and policy variables to enhance or curb the expansion of bubble in real estate markets.

Theoretically, asset price bubbles are defined as the difference between the actual price and the fundamental value of the asset driven by speculative demand. The fundamental value of the asset is equal to its sum of discounted stream of dividends. In mainstream macroeconomics,

economists focus on bubbles that are consistent with rational expectation general equilibrium. These bubbles are called rational bubbles.

According to the long-listed literature on bubbles, two conditions are required for bubbles to be rational. First, bubbles must provide sufficient return compared to that of investment. Second, bubbles must not outgrow the economy. In other words, rational bubbles must grow fast enough in order to be worth holding, but not too fast such that the economy cannot afford eventually.

Therefore, the economy that allows rational bubbles to exist must have the interest rate lower than the growth rate of economy. That is, the economy must either suffer from over-investment due to the lack of financial instruments, or get stuck with credit constraint. Such financial imperfection happens to realistically appear in every economy, especially emerging markets.

In credit constrained economy, the existence of rational bubbles leads to the booms in both consumption and investment. The reason is that rational bubbles play a crucial role as the additional collateral and relax credit limit. The rapid expansion of credit provision in turn fuels bubbles and put the entire economy in the risky boom.

The above insight of bubbles is obtained from the long-listed theoretical works, for example see Tirole (1985) and Weil (1987), and Santos and Woodford (1997) Caballero and Krishnamurthy (2006), Caballero, Farhi and Hammour (2006), Kocherlakota (2009), Farhi and Tirole (2012), Martin and Ventura (2011), Ventura (2012), Hirano and Yanagawa (2013), Bejan and Bidian (2015), Werner (2015), Miao and Wang (2015), Miao, Wang, and Zhou (2015). Their contributions are to shed light on how bubbles emerge and correlate to other economic variables. Yet, it happens that the nature of bubbles itself relies on the indeterminacy of the equilibrium. Thus, the theoretical work does not really guide us how we can empirically calculate bubbles in the real world.

However, economists have attempted to build up methods for the bubble-oriented crisis detection. The methods that have been used in the literature may be categorized as: (1) analyses of various ratios that typically compare house prices to either rents or incomes (Himmelberg, Mayer, and Sinai (2005)); (2) regression analyses of various sorts, including models based on either housing supply and demand theory or asset pricing (present value) concepts as well as

cointegration tests (Abraham and Hendershott (1996), Black, Fraser, and Hoesli (2006), and Oikarinen (2009)); and (3) a method drawn from physics that focuses on the rate of growth in prices (Zhou and Sornette (2006)).

Since 2011, Union Bank of Switzerland (UBS) have constructed the UBS Swiss Real Estate Bubble Index by comprising several indicators; see Holzhey (2013). The six indicators from the Swiss real estate market are applied, i.e. i) Own home price relative to annual rent; ii) Home price relative to household income; iii) Construction sector relative to gross domestic product; iv) Own home prices relative to consumer prices; v) Mortgage volume relative to income; and vi) Credit applications for residential property not intended for owner occupancy (UBS clients). The principal component analysis is used to extract the bubble content from these indicators. This method provides an interesting approach to develop real estate bubble index from several indicators and comprise into one single index that makes it easy to track bubble situation in any given market.

In detail, the literature on rational bubbles has advanced greatly over the past three decades. Tirole (1985) and Weil (1987), and Santos and Woodford (1997) have shown that bubbles can emerge as stores of value to help solve the shortage of financial instruments in the overlapping-generations model. In a dynamically inefficient economy, bubbles absorb savings out of inefficient investment to raise the rate of return. Thus, bubbles are favorable. However, Tirole's results contradict empirical evidence which shows that most economies with bubble episodes are in fact dynamically efficient and that bubble booms are often accompanied by investment booms (see, e.g., Abel, Mankiw, and Zeckhauser, 1989).

Following studies of the balance-sheet effect of credit constraints, such as Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), many studies have examined how bubbles can exist in a dynamically efficient economy and how bubbles crowd in investment (see, e.g., Caballero and Krishnamurthy (2006), Caballero, Farhi and Hammour (2006), Kocherlakota (2009), Farhi and Tirole (2012), Martin and Ventura (2011), Ventura (2012), Hirano and Yanagawa (2013), Bejan and Bidian (2015), Werner (2015), Miao and Wang (2015), Miao, Wang, and Zhou (2015)). Although different models have different details, they share the same key element: financial imperfection. In the dynamically efficient economy, limited pledgeability generates a credit constraint, which suppresses the demand for capital. As a result, the rate of

interest falls below the growth rate of the economy. This creates room for bubbles. If the economy is initially credit constrained, bubbles can act as collateral to relax the credit constraint and raise the demand for capital. Thus, bubbles can crowd in investment.

Recently, Martin and Ventura (2012, 2015a, 2015b) have made important progress in rational bubble modeling. They introduced so-called bubble creation. That is, in every period, new bubbles can be created out of nothing as long as they are consistent with the prevailing belief of agents. Throughout the present paper, this bubble-creation belief is also referred to as market sentiment. Bubble creation helps to relax the bubble no-arbitrage condition and notably enlarge the bubbly equilibrium feasible set. Martin and Ventura (2012, 2015a, 2015b) have also shown that there is an optimal bubble creation that provides maximum investment which can be achieved by fiscal intervention to transfer wealth across time.

Sornette (2003) has pointed out that financial crashes are outliers and have different distribution from the normal price increment. In other words, the distribution is still smooth but consists of two populations which are the body and the tail. The tail is more disperse than the body. Specifically, empirical works point out that the Gaussian and stretched exponential distributions respectively fit the body and the tail well. Bourassa, Hoesli, and Oikarinen (2016) have compared various methods of measuring bubbles and conclude that simple price-rent ratio is the effective indicator.

Even though, there are number of research investigating the cause of bubble and its relationships to macroeconomic variables, e.g. investment, interest rate, output growth, and credit to private sector, the empirical studies in real estate bubble are still limited due to the lack of the indicators to differentiate the real estate bubble from the increase of the house price due to the real demand. Therefore, empirical literature related to housing bubbles usually focus on the relationship between housing price and macroeconomic factors. In the cross-country study, Tsatsaranis and Zhu (2004) have investigated the cross-country determinants of housing price dynamics using the VAR method. The results suggest that the inflation is the importance driver of the house price. In addition, bank credit, short-term interest rate and interest rate spread also have strong influence due to their role in mortgage finance. In later study, the role of monetary policy is usually underscored. Goodhart and Hofmann (2008) have investigated the link between house price, monetary policy variable (money and credit) and other macroeconomic factors. They

showed that there is a strong linkage between house prices and monetary variables during 1985 to 2006 when housing price boom is found in the US. Therefore, the effects of shock in money and credit are stronger during the booming period. Moreover, shocks to GDP, inflation and interest rate also have significant impact on housing price. Furthermore, Bordo and Lane (2014) have calculated the historical decomposition of housing price boom in the developed countries. Using panel VAR, they found that loose monetary policies, low inflation and easy bank credit can explain the rising housing price during the boom period. However, in some episodes of housing boom, i.e. housing bust of U.S. market in 2006 cannot be explained by the bank credit because the role of the shadow banking system to provide easy credit was more prominent than the formal banking system. Panagiotidis and Printzis (2015) have examined the role of housing market in Greece. They found that mortgage loan and inflation are the main factors that influence house price in short run. However, the retail trade plays the most significant role in the long-term determinant. (See Joebges, Dullien and Marquez-Valazdues (2015) for the literature review in the determinant factors of housing bubbles in both theoretical and empirical perspective.)

The dynamic feedback linkage between the housing bubbles, monetary policy and macroeconomic factors is emphasized in the theoretical literature and the empirical literature of housing price determinants. However, there is the lack of research that focuses on the housing bubble in the empirical perspective due to the lack of indicators for bubbles. Moreover, the current literature is also focused on the industrial countries. To the best of our knowledge, the association between the housing bubbles and macroeconomic variables has not been investigated in the emerging market. Therefore, the results from this research can provide important contribution to the literature.

In case of Thailand, the real estate bubble is usually mentioned as one the reason of the problems leading to the Asian financial crisis in 1997. Since then, Real Estate Information Center has been established to collect data in real estate market and monitor current market condition. However, there is still lack of single indicators that can be used as a proxy for the real estate bubble, which makes it difficult to monitor the current condition of the Thai real estate market. Therefore, the development of the indicators for real estate bubble in Thailand is crucial, not only for monitoring the bubble condition in real estate but also for examining the relationship between bubbles and other key macroeconomic variables including the monetary policies. Interestingly,

Luengaram and Thepmongkol (2022) theoretically have shown that the macro-prudential monetary policy like loan-to-value ratio is effective against bubbles in the economy with low financial development like Thailand.

2. Rational Bubble Theories

Kikuchi and Thepmongkol (2020) have outlined the simplified model for the bubbles in the price of the productive asset in a small open economy. Denote r^* is a constant world's interest rate where the interest rate of the small open economy is assumed to be equal to, x is the stock of productive asset, p is the price of asset, $I(\cdot)$ is a function of new investment on asset, $d(\cdot)$ is a function of asset's dividend, and θ is the depreciation rate.

In equilibrium, the economy follows the law of motion of asset and the no-arbitrage condition between investing in an asset and lending accordingly:

$$(1) \quad x_{t+1} = (1 - \theta)x_t + I(p_{t+1})$$

$$(2) \quad [(1 - \theta)p_{t+1} + d(x_t)]/p_t = 1 + r^*$$

From the re-iteration of (2), the price of asset can be decomposed into two elements which are fundamental value and bubbles:

$$(3) \quad p_t = \underbrace{\frac{1}{1+r^*} \sum_{i=0}^{\infty} \left(\frac{1-\theta}{1+r^*}\right)^i d(x_{t+i})}_{\text{fundamental}} + \underbrace{\lim_{k \rightarrow \infty} \left(\frac{1-\theta}{1+r^*}\right)^k p_{t+k}}_{\text{bubbles}}$$

The first term of the right-hand-side of (3) is defined as fundamental price which is the sum of discounted stream of asset's dividends. The difference between the current price and the fundamental level is then bubbles. It is shown that bubbly equilibrium occurs as a possible choice of initial current price that can possibly be set above the fundamental level. In other words, the nature of bubbles is pure indeterminacy of equilibria.

Figure 1: Fundamental equilibrium vs bubbly equilibrium

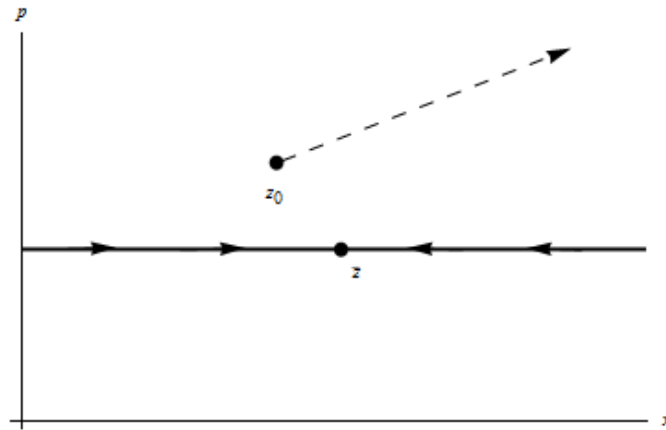


Figure 1 summarizes the result that there is the fundamental equilibrium stable manifold (solid line) where the dynamic on the manifold converges to the steady state (point $\bar{z}$). However, when the initial price is set above the fundamental manifold (point z_0 above solid line), the economy is on the bubbly equilibrium where the price is eventually on the explosive path and can be suddenly crashed on to the fundamental manifold at any point in time with a given probability.

The work of Kikuchi and Thepmongkol (2020) lacks the aspect of endogenous interest rate for the sake of tractability. Other existing works like Tirole (1985) suggest that in the environment that interest rate is too low compared to the real economic growth, perhaps due to lack of financial instrument, bubbles can emerge to raise the economy's rate of return while bubbles can be rationally maintained.

Figure 2: Crowding-out-investment bubbles with endogenous interest rate

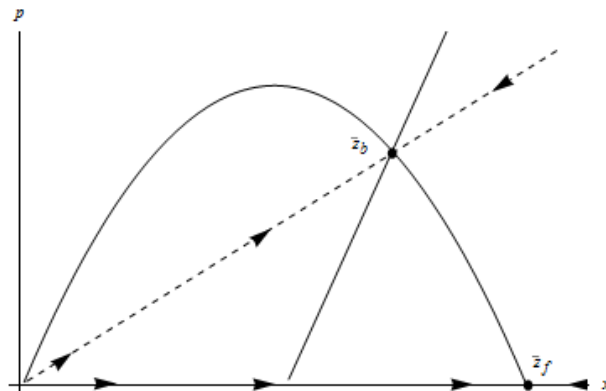
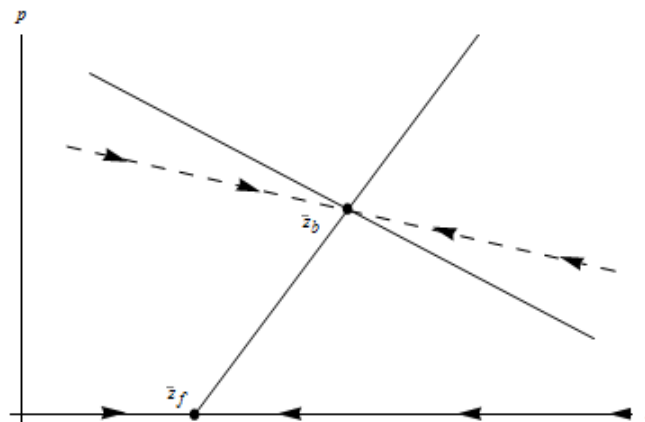


Figure 2 shows the phase diagram where the fundamental price of this asset is at zero and the horizontal axis forms fundamental stable manifold with fundamental steady state $\bar{z}_f$. There exists the other steady state called bubbly steady state $\bar{z}_b$ where all points on the saddle path (dashed line) converge to. In detail, given the initial capital stock at $\bar{z}_f$, the equilibrium may switch to the bubbly one and converge to $\bar{z}_b$. These bubbles crowd out investment as they compete for savings to solve the fundamental overinvestment problem.

Figure 3: Bubbles crowd in investment



The crowding-out effect is not universal feature for bubbles. Many recent works have shown that bubbles can crowd in investment. For example, Farhi and Tirole (2012) shows that bubbles that emerge because of the underlying credit constraint problem can crowd in investment. The logic is that the existing credit constraint suppresses the demand for loan and hence results in low interest rate fundamentally. Bubbles act as additional collateral to expand the credit limit leading to more credit provision and investment. Figure 3 illustrates this dynamic.

The boom-bust episode of bubbles captures the sudden switch between bubbly equilibrium and fundamental equilibrium. During the bubble boom, prices of goods and services obviously increase. GDP rises.¹ Thus, consumption also rises. The capital inflow and credit provision expand. Investment can theoretically be either increasing or decreasing depending on the underlying economic problem, although most empirical studies support the co-occurrence between bubble boom and investment boom. Interest rate might slightly be tricky as theories say

¹ Note that bubbly assets are counted in GDP as well.

bubbles help raise interest rate in comparison to fundamental equilibrium. However, within the bubbly dynamics, Figure 2 and 3 suggest that interest rate decreases while bubbles are booming. Table 1 summarizes the relationship.

Table 1: Expected dynamics during bubble booms

Macroeconomic indicators	Expected dynamics during bubble boom
Price of bubbly asset	Increasing
Inflation	Increasing
GDP	Increasing
Consumption	Increasing
Investment	Increasing or decreasing
Capital outflow	decreasing
Interest rate	Decreasing
Macro-prudential Policy	Decreasing

3. Real Estate Bubble Indicator

In this chapter, the Thailand's real estate bubble index (RBI) is computed. We start from the introduction of the principal component analysis method, proposed by UBS to construct the real estate bubble index. Next, the real estate data in Thailand is discussed and the sub-indices representing each component of real estate bubbles are identified. Finally, the real estate bubble index (RBI) in Thailand is calculated. We also discuss the validity of our proposed index.

3.1 The UBS methodology

In this part, we outline the statistic method suggested by UBS. The Principal Component Analysis (PCA) is useful for this purpose. The PCA method is technique to reduce variables of interest by generating set of new variables as a linear function of the original ones. The new variables are generated in order to include the variation of original variables. We call new variables as the i^{th} order principal components (PCA_i). In addition, the PCA method is also applied in factor

analysis to extract the latent factors from the set of observed variables². Based on this property, the real estate bubble is viewed as the unobserved factors that can influence the movement of several observed variables related to the unbalanced of demand and supply in real estate market. Therefore, we also apply the PCA technique to develop the real estate bubble index in this study. The process of generating the real estate bubble index using the principal component is explained as follows.

The first principal component (PCA_1) is calculated as a linear function of the original p variables, written as follows.

$$PCA_1 = w_{11}Z_1 + w_{12}Z_2 + \dots + w_{1p}Z_p \text{ or } PCA_1 = w'_1z \quad (2)$$

$$w = [w_{11} \ w_{12} \ \dots \ w_{1p}] , z = [Z_1 \ Z_2 \ \dots \ Z_p]$$

The first principal component includes the variation of these p variables. Therefore, we calculate the weight matrix (w) under the condition that maximize $Var(w'_1z)$ with constraint that $w'_1w_1 = 1$.

Next, we generate the second principal component (PCA_2) as a linear function of the original p variables capturing the remaining variation. Thus, PCA_2 should have not got correlation with PCA_1 .

$$PCA_2 = w_{21}Z_1 + w_{22}Z_2 + \dots + w_{2p}Z_p \text{ or } PCA_2 = w'_2z$$

where w_2 are calculated to maximize $Var(w'_2z)$ with $w'_2w_2 = 1$, $w'_1w_2 = 0$ and $Cov(w'_1x, w'_2z) = 0$.

Next, we construct $PCA_3, PCA_4, \dots, PCA_{p-1}, PCA_p$ using the same procedure as PCA_2 .

In the case of Thailand, the availability of data and set of variables for constructing the real estate bubble index are discussed in the next section. These variables are selected to represent the unbalanced demand and supply in the real estate market. This imbalance makes the

² See Stock and Watson (2005, 2016) for the detail on the application of the PCA method to estimate the latent variables in the factor models.

market price deviate from the fundamental value driven by speculative demand, and hence is embedded with bubbles.

Notably, the Z-score technique is used to standardize each variable before conducting PCA:

$$s = \frac{x - \bar{x}}{\sigma}$$

3.2 The data on the real estate market in Thailand

Based on the literature and availability of data in the case of Thailand. We collect the following data for the construction of each component of the indicators representing the real estate bubbles. The lists of related variables are shown in Table 2.

Table 2 Real estate data in Thailand

Variable	Data	Source	Remark
1. Town House Price	a) House Price Index: Town House (quarterly, discontinued) b) House Price Index: Town House (monthly)	a) Government Housing Bank b) Bank of Thailand	Average monthly data of data b) to obtain quarterly data and then extend it by adding data a)
2. Population	Population: Whole Kingdom (yearly)	Department of Provincial Administration	Use interpolated with quadratic trend method
3. Number of Household	Number of Household: Whole Kingdom (yearly)	National Statistical Office	Use interpolated with quadratic trend method

3. Gross Domestic Product (real and nominal GDP)	Real GDP (GDP chain value measure at 2002 price) Nominal GDP (GDP at current market prices) Both are based on quarterly and seasonal adjusted data	National Economic and Social Development Board	quarterly data
4. Consumer Price Index	CPI (monthly)	Bureau of Trade and Economic Indices	Average to obtain quarterly data
6. Real estate – related sectors	Real GDP: Non-Agri: Construction plus Real Estate Activities (seasonal adjusted, quarterly)	National Economic and Social Development Board	quarterly data
7. Newly-launched Unit	Housing Completed: Bangkok and Vicinities (monthly)	Real Estate Information Center	Sum up to obtain quarterly data
8. Mortgage Volume	a) Property Credit: Commercial Bank Personal Housing Credit (quarterly)	Bank of Thailand	quarterly data

Next, we use these data to generate the indicators of each parts of real estate bubbles. The details on the calculation of each variable are listed as follows.

i. Indicators for the demand growth (change in demand of real estate in relative to the income growth)

Proxy variable Z_{11} : Change in Town House Price index to Household Income

$$Z_1 = \Delta \frac{\text{HousePriceIndex:TownHouse}}{\text{PerCapitaGDPIndex}}$$

ii. Indicators for the price pressure (change in price of real estate in relative to inflation)

Proxy variable Z_2 : Town House Price index to Overall Consumer Price Index:

$$Z_2 = \frac{\text{Change in Town House Price Index}}{\text{Consumer Price Index}}$$

iii. Indicator for financial sectors (mortgage growth in relative to nominal GDP)

Proxy variable Z_3 : Commercial Bank Personal Housing to nominal GDP

$$Z_3 = \frac{\text{PropertyCredit: Personal housing (CommercialBank)}}{\text{Nominal GDP}}$$

iv. Indicator for the production (supply side) in the macroeconomy (construction plus real estate activities in relative to total output)

Proxy variable Z_4 : Construction to Gross Domestic Product:

$$Z_4 = \frac{\text{Construction} + \text{Real Estate Activities}}{\text{Real GDP}}$$

v. Indicator for the supply growth (Change in supply of real estate in relative to quantity household)

Proxy variable Z_5 : Newly-launched unit to Quantity household:

$$Z_5 = \frac{\text{New Launched Unit}}{\text{Quantity Household}}$$

All of the five variables are shown in Table 8 in Appendix. They are applied as a ratio between the indicators in housing market, i.e. change in house price, demand for mortgage,

demand for construction, newly launched unit, and their fundamental factors (household income, quantity household, consumer price, gross domestic product and quantity household, respectively).

The real estate bubble index (RBI) is computed as followed.

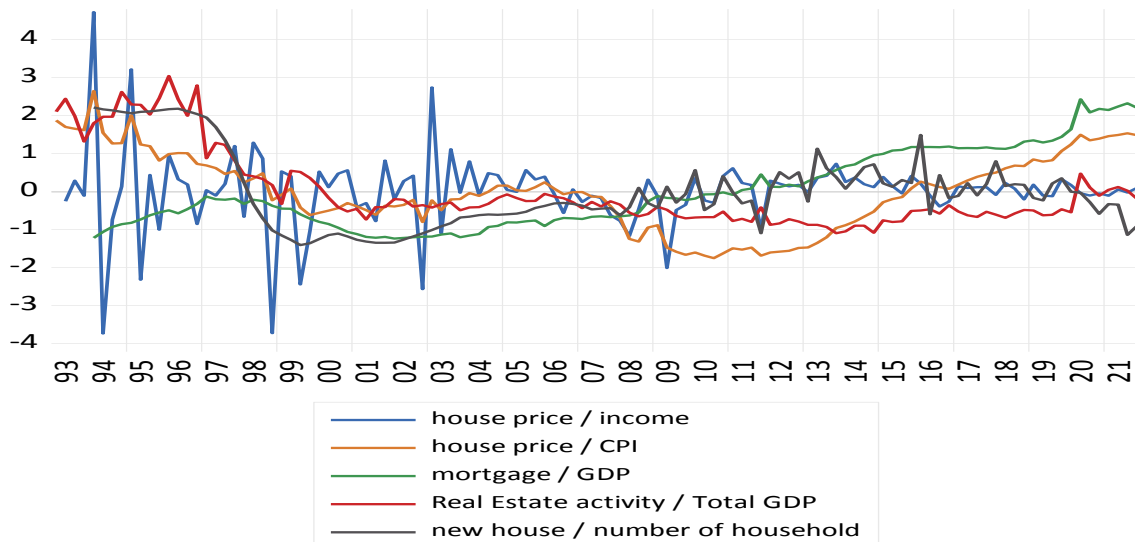
$$RBI_t = w_1Z_1 + w_2Z_2 + w_3Z_3 + w_4Z_4 + w_5Z_5$$

We assign the weights from the principal component estimation. To select that which principal component ($PCA_1, PCA_2, \dots, PCA_5$) to be used to compute the real estate bubble index, we look at the correlation between each principle components and the other indicators of the real estate market and macroeconomic variables, i.e. Thai Housing Developers Sentiment Index (HDSI), property sectorial index from stock exchange of Thailand, and policy interest rate, where all are adjusted by inflation to be in real term.

3.3 Constructing the real estate bubble index in Thailand.

This section proposes the results of real estate bubble index in Thailand. First, we compute each of sub-indices for real estate bubble. Five indicators are computed based on definition and data outlined in section 3.2. The data had been standardized and shown in Figure 4.

Figure 4 Sub-indicators for real estate bubble index



Considering the movement in each of five sub-indicators for real estate bubble index, most of the sub-indicators are extremely high before the Asian crisis in 1997 (the standardized value are higher than two). The launch of new house (sub-index 5) and the ratio of real estate activity (Sub-index 3) are two main sources of real estate bubble before 1997. The role house price change (Sub index 1) is fluctuated but still in the high level as well. For the house price per CPI (sub-index 2), even though the house price level is high during 1993 to 1995, house price gradually declines before the outbreak of crisis in 1997. Finally, the role of mortgage growth is minimal as the sub-index 3 is well below the average level before 1997.

After 1997, all sub-indicators of real estate bubble decline significantly, the new house and mortgage stay at the low level (below -1) until 2003. During the period between 2003 and 2007, all indicators seems to stabilize as the indices' values slowly increase and are close to zero. After the Global Financial Crisis in 2007, the house price per CPI (sub-index 2) and the real estate activity (sub-index 4) are significantly weakening and remain frail until 2102. The house price change (sub-index 1) also drops but quickly rebounds since 2009. The new house index (sub-index 5) and the mortgage indicator (sub-index 3) remain stabilized during 2007 to 2012.

Recently after 2012, the mortgage (sub-index 3) and house price per CPI (sub-index 2) indicators demonstrate an increasing trend. The real estate activity (sub-index 4), which stays at the low level since 1997, increases during the last two years (2020-2021). On the other hand, the new house indicator decreases during the same period (2021).

Overall, the movement of sub-indicators clearly specifies the bubble period before the Asian Crisis in 1997, followed by the bubble burst, which shows the significant decline in most indicators. After 2002, the degree of bubble increases but the global financial crisis in 2007 slows down the movement in some sub-indices. However, recent developments in some sub-indicators during 2020 to 2021 provide the sign of accumulation in bubbles.

We then apply the Principal Component Analysis to extract the bubble component from these sub-indicators and compute the real estate bubble index using the method outlined in Section 3.1. We compute the correlation matrix for five sub-indicators. The results are shown in Table 3.

Table 3 Correlation coefficients of five sub-indicators for real estate bubbles

	House price / Income	House price / CPI	Mortgage	Real estate sector activity	New house
House price / Income	1.00				
House price / CPI	0.18	1.00			
Mortgage	0.01	0.25	1.00		
Real estate sector activity	-0.01	0.61	-0.27	1.00	
New house	0.11	0.38	0.09	0.64	1.00

The results in Table 3 show that the new house and the real estate activity has the highest degree of correlation (0.64) followed by the real estate activity and the price per CPI (0.61). The indicator for price change seems to have lowest degree of correlation with other sub-indices. Most of sub-indices have positive pairwise relationship, with the exemption for the mortgage and the real estate activity.

We first consider the table for the proportion of variation in the group of five sub-indicators explained by each principal component. The results from the eigenvalue show that the first eigenvalue can be applied to explain the 42.35 percent to total variation. The second eigenvalue can be accounted for 23.91 of variation. In sum, these two components are accounted for 66.45 percent, which is the major part of the total variation.

Based on the statistics from Table 4, the first eigenvalue can be applied to compute the principle component to explain the real estate bubbles. However, we also calculate the second principle component to compare and examine the most suitable indicators for the real estate bubbles. The factor loading can be obtained from the eigenvectors for each principle components. The results of eigenvectors are shown in Table 5.

Table 4 The five eigenvalues computed from the group of real estate bubble indicators and the proportion in the variation of data

Eigenvalues: (Sum = 5, Average = 1)						
Number	Value	Difference	Proportion	Cumulative		Proportion
				Value	Proportion	
1	2.117509	0.921902	0.4235	2.117509	0.4235	
2	1.195606	0.220478	0.2391	3.313115	0.6626	
3	0.975128	0.378251	0.1950	4.288243	0.8576	
4	0.596878	0.481999	0.1194	4.885121	0.9770	
5	0.114879	---	0.0230	5.000000	1.0000	

Table 5 The five eigenvectors for each of principal components.

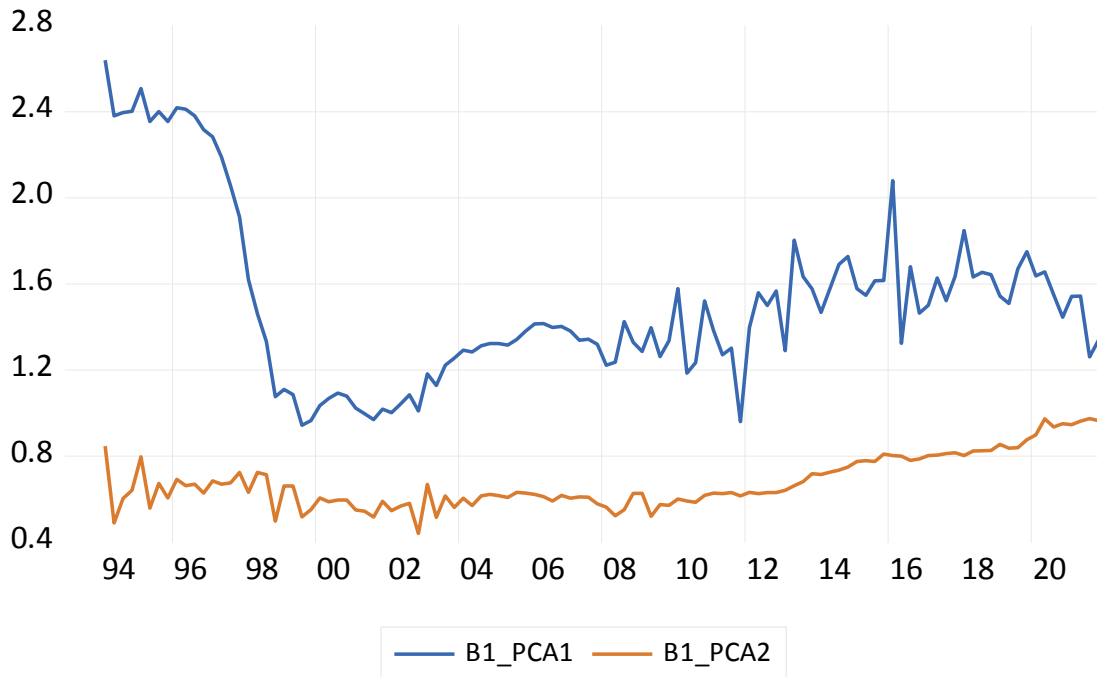
Variable	PCA1	PCA2	PCA3	PCA4	PCA5
Z1: PRICE_INCOME	0.138218	0.310169	0.928087	0.063088	0.139156
Z2: THPRICE_CPI	0.550529	0.277398	-0.061754	-0.630678	-0.467332
Z3: MORTGAGE_HH	0.020482	0.843209	-0.351552	0.140874	0.380979
Z4: CONS_GDP	0.610963	-0.340240	-0.073499	-0.127470	0.699507
Z5: NEW_HH	0.551469	-0.009036	-0.076479	0.749781	-0.357464

Hence, we compute the first and second principle components using the following equations and the results are shown in Figure 5.

$$PCA1 = 0.138 * S1 + 0.551 * S2 + 0.020 * S3 + 0.611 * S4 + 0.551 * S5$$

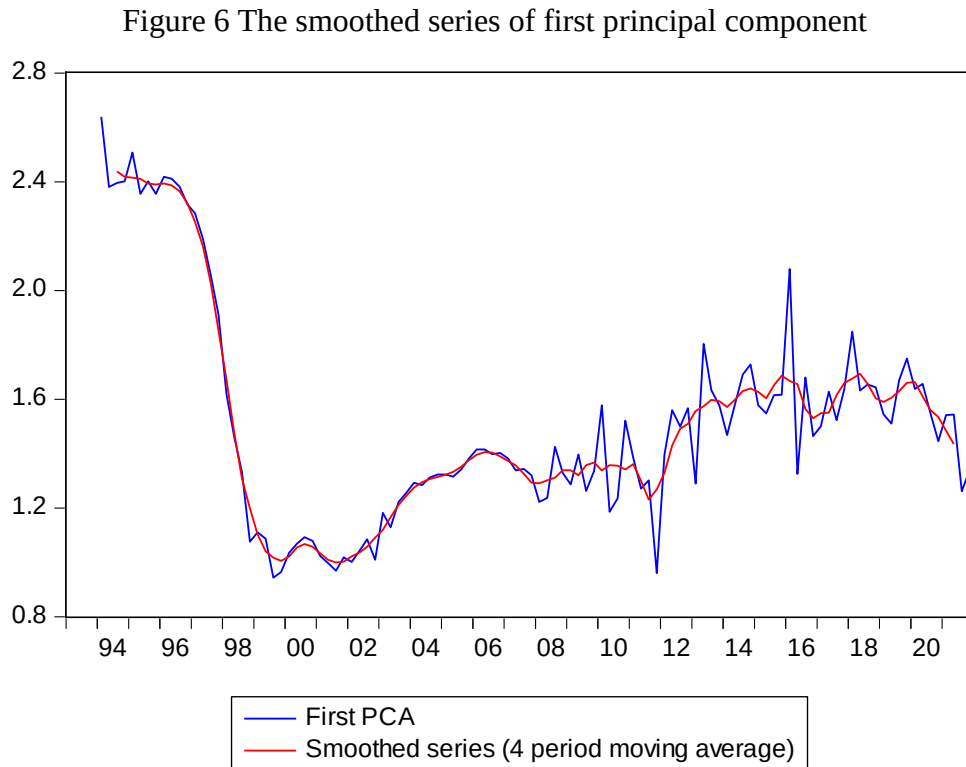
$$PCA2 = 0.310 * S1 + 0.278 * S2 + 0.843 * S3 - 0.340 * S4 - 0.009 * S5$$

Figure 5 The first and second principal components from the group of five real estate bubble indicators



From Figure 5, the first principle component (PCA1) explains episode of the real estate bubbles in Thai economy before the Asian crisis (1994 to 1996) and the bubble burst during the crisis period (1997 to 1999) perfectly. The PCA1 series gradually increases between 2000 to 2005, representing recovery phrase in the Thai housing market. The condition in real estate marketing gradually improves. Hence from the PCA1 during the Global Financial Crisis (GFC) in 2007 to 2009, the Thai economy has a short period of recession followed by the great flood in 2011. The PCA1 fluctuates during those periods and gains the upswing process again after 2012. After 2017 the PCA1 gradually declines. The Thai economy continues to slow down from the export uncertainty and the Covid-19 pandemics. Concurrently, the PCA1 displays the pattern of sharp droplet after 2020. Overall, the PCA1 series provides a good proxy to explain the development and balance of demand and supply in the real estate market in Thailand. In contrast, the second principle component (PCA2) series shows the more stable pattern especially before 1997. As a result, the PCA2 series does not show the sign of the well-known real estate bubbles in Thailand before the Asian financial crisis.

Hence, we focus on the first principal component (PCA1). We follow the suggestion from the UBS methodology to compute the moving average of the series computed from the principal component method to smooth the effect of random shock that may temporarily influence each component of the RBI. Hence, we employ the 4-period (quarter) moving-average center filtering to smooth the series. The results are displayed in Figure 6.



As can be seen from Figure 6 the smoothed series still preserves the pattern of development that track the conditional of real estate market in Thailand before and after the financial crisis, the GFC, and the great pandemic period. However, the movement pattern is smooth, which could be the better proxy the development in real estate market and clean out the provisional noise in each component. Hereafter, we apply the moving average series of the first principal component as the real estate bubble index (RBI) in Thailand.

3.4 Checking for validity of the real estate bubble index

In this section, we consider the validity of RBI by comparing with the related variables, i.e., the Housing Developer's Sentiment Index (HDSI) and the property development sectorial

index in stock exchange of Thailand (PROP). We also standardize these series. The data of these variables are shown in Table 9 in Appendix and the time series graph and correlation coefficients are shown here in Figure 7.

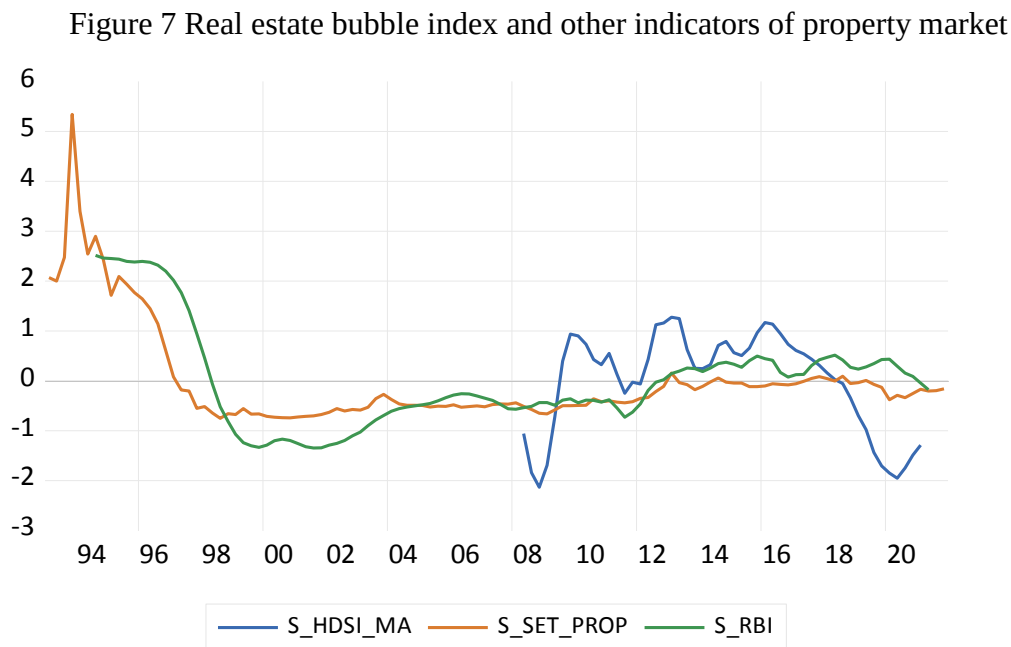


Figure 8 The intertemporal cross-correlations between real estate bubble index (RBI) and housing development sentiment index (HDSI) or property development sector index (SET_PROP)

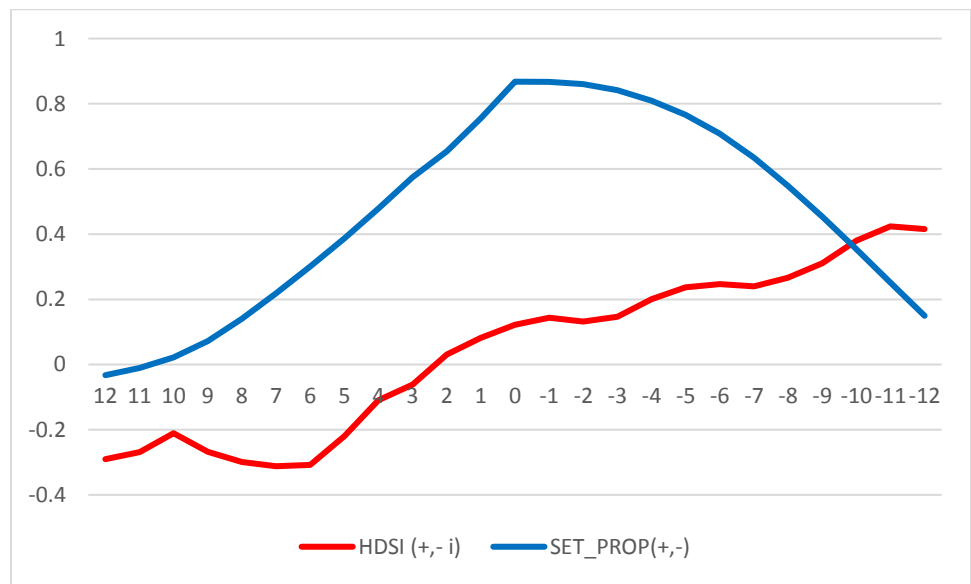


Table 6 Correlation coefficient among real estate bubble index (RBI), Housing Developer's Sentiment Index (HDSI) and stock market index for the property development sector (SET_PROP)

(P-value)	RBI	S_HDSI	S_SET_PROP
RBI	1.000000 -----		
S_HDSI	0.121644 (0.3763)	1.000000 -----	
S_SET_PROP	0.845536 (0.0000)	0.341057 (0.0108)	1.000000 -----

Note: HDSI and SET_PROP are standardized series. The value in the parenthesis below correlation coefficient is p-value for testing significance of this correlation between variables.

We first consider the stock market index for the property development sector. Under the rational expectation, the stock prices reflect the rational expectation of investors to the fundamental and risk factors. The results from Figure 7 show the pattern of concurrent movement between the RBI and SET_PROP. Moreover, the correlation coefficient from Table 6 shows the high degree of correlation (0.84) between RBI and SET_PROP. In case of the Housing Developer's Sentiment Index (HDSI), the degree of contemporaneous correlation is low (0.1216) and statistically insignificant. However, Figure 7 shows that the co-movement pattern between RBI and HDSI may exhibit the lead-lag relationship pattern. Furthermore, we explore the possibility of the lead-lag relationship between these variables. The intertemporal cross-correlations are expressed in Figure 8.

Figure 8 shows the degree of correlation between the real estate bubble index (RBI) and the SET_PROP (HDSI) in blue and red lines respectively. The period in the X axis represent the concurrent (period = 0), leading (period = -1 to -12) and lagging indicators (period = 1 to 12) of these variables to RBI. First, we consider the cross-correlation between RBI and SET PROP (blue line), the pattern in Figure 8 indicates the concurrent relationship between RBI and SET_PROP is strongest. Moreover, the pattern of leading indicator of SET_PROP to RBI is stronger than those of the lagging relationship. In the case of housing development sentiment index (red line), which

develop from the questionnaire to entrepreneur in property development firms. This index is constructed to represent the positive expectation of the business sector. From Figure 4, the movement in HDSI is in the similar long-term trend to that of the RBI. However, HDSI has more fluctuation than the RBI. Hence, the correlation coefficient is around 0.13. The HDSI shows the significant positive relationship with RBI as a leading indicator. On the right side of the cross-correlation graph, i.e., the leading indicators, the intertemporal correlation is positive and increases over the long horizon.

As a result, we find the linkage between each proxy of the expectation in the real estate market. However, the existing indicators have their own disadvantage. The SET_PROP could be affected by the systematic risk in the stock markets. The HDSI index has high fluctuation and data is available only after 2008. Hence, our proposed index (RBI) could be useful index to monitor real estate market in Thailand. Moreover, the RBI could also be used as the proxy to investigate the response of real estate bubble to the movement in the macroeconomic fundamental factors. In the next chapter, we investigate this relationship using the Vector AutoRegression (VAR) framework.

4. The Responses of Real Estate Bubbles to Key Macroeconomic and Policy Variables

This chapter analyzes factors affecting the movement of real estate bubble in Thailand. Using the RBI as a proxy, we apply the quantitative analysis based on a Bayesian Vector AutoRegressive (BVAR) model. Based on theoretical framework outlined in Chapter 2. We identify the variables grouped into three categories, i.e., macroeconomic fundamentals, financial markets and monetary policy variables. The methodology and empirical results are discussed as follows.

4.1 Econometric methodology

This section explains a BVAR model, which is the main methods for analyzing the dynamic relationships between the real estate bubble index and the key macroeconomic, financial market, and monetary policy variables. We use the data from the first quarter of 1993 to the fourth quarter of 2021 corresponding to the range of RBI in the last chapter.

. The key macroeconomic variables consist of Gross Domestic Product (GDP) and the component in aggregate demand, i.e., consumption and investment. In addition, consumer price index is included as a proxy for inflation. For financial market data, we employ the ratio of private credit to GDP, capital flow (inward foreign portfolio investment) and interest rate (minimum loan rate - MLR). Finally, we consider the two dimensions of monetary policies, i.e. the repurchase rate are applied as the proxy of conventional monetary policy, and the loan to value (LTV) ratio, which is currently the key instrument in the macro-prudential policies. We also include the two risk factors as the exogeneous variables in the VAR models to control the effects of other risk factors. The global economic policy uncertainty (GEPU) proposed by Baker et al.(2016) is applied to represent external risk factor. The Thailand's Political Uncertainty Index (PUI) (Luangaram and Sethapramote, 2018) is used as a proxy for domestic risk factor. See Table 7 for summary of all variable notations.

Table 7 The lists of variables in the model and data sources.

Abbreviation	Variable	Source	Unit
RBI	Real Estate Bubble Index	Author Calculation	Standardized index
GDP	Gross Domestic Product at constant price (S.A.)	NESDC	Millions of baht in logarithm
CONPG	Consumption Expenditure (private and public) at constant price (S.A.)	NESDC	Millions of baht in logarithm
INVP	Investment (gross fixed capital formation) at constant price (S.A.)	NESDC	Millions of baht in logarithm
MLR	Minimum loan rate	Bank of Thailand	Percent
CREDIT	Ratio of private credit to GDP	Bank of International Settlement (BIS)	Percent

FPI	Foreign Portfolio Investment (FPI) inward -net	Bank of Thailand	Million of US dollar
RP	1-day repurchase rate	Bank of Thailand	Percent
LTVR	Inversed Loan-to-Value ratio	Bank of Thailand	Percent
EPU	Global Economic Policy Uncertainty Index	www.policyuncertainty.com	Index
PUI	Thailand's Political Uncertainty Index	Luangaram and Sethapamote (2018)	Index

A Bayesian framework is applied for estimating the interaction between variables in this study. The over-parameterization in the VAR model, high fluctuation and limited length of data in real estate bubble index make it difficult to estimate the parameters in the VAR model and affect the accuracy in computation of the impulse response function. Therefore, we use the BVAR model in this study. The BVAR model put the prior information in estimation procedure. Hence, we can expect the improvement in model accuracy when the degree of freedom of model is small. We select a two-lags VAR model based on the Akaike Informational Criteria (AIC)³. The generalized impulse response function (GIRF) is applied to calculate patterns of responses of macroeconomic variables to shocks. We include 95 percent and 90 percent posterior coverage band in the impulse response function to provide assessment tools for the significance of responses to shocks. We obtain the posterior distribution via is Markov-Chain Monte Carlo (MCMC) algorithms.

Empirical results of this paper consist of the BVAR models for the fundamental factors and the real estate bubbles. We start from analyzing the role of macroeconomic variables in reinforcing the bubble in real estate market. Hence, we first set up Model 1 with five variables. The first three variables are macroeconomic, i.e. Gross Domestic Product (GDP), Inflation (CPI), Minimum Loan Rate (MLR). We also include Credit-to-GDP ratio (CREDIT) as the proxy for financial liquidity. Finally, the RBI is included. We also consider the two separated sub-models.

³ The alternative number of lags in a VAR model is also considered. The results show that changes in number of lags do not affect the general results.

First, Model 1a substitutes the GDP for the total consumption expenditure. Secondly, Model 1b uses investment in place of the GDP in the estimation. Next, we consider the role of financial markets and monetary policy variables. The Model 2 consists of the Foreign Portfolio Investment (FPI), repurchase rate (RP), loan to value ratio (LTV), and real estate bubble index (RBI). Both models 1 and 2 include global economic policy uncertainty index (EPU) and Thailand's political uncertainty index (PUI) as exogenous variables to control for the external and internal risk factors, respectively.

To examine what causes the real estate bubbles in Thailand, the impulse response functions are computed. We consider the responses of the RBI to shocks in other factors in the models. The results are displayed and discussed in the following section.

4.2 Empirical results

We first consider the impulse response from the Model 1A. The responses of RBI to all other variables in the model are computed. Moreover, we also include the impulse response from model 1B and model 1C for the variables of total consumption and investment. The results are displayed in Figure 9.

We first consider the role of total output (GDP) and inflation (CPI) to the bubble formation in the Thai real estate markets. Interestingly, the sign of response of the RBI to each of macroeconomic variable is consistent with the theoretical framework in Chapter 2. The RBI increases when there is a positive shock in total output (GDP), consumption, investment and inflation. The impulse response shows that the RBI gradually increases in the uptrend business cycle when the GDP and inflation increase concurrently. In particular, the size of response is small. The effect of inflation is significant at 10 percent level, while the effects of GDP is insignificant. Even though the magnitude of response on RBI to a one standard deviation shock in consumption is higher than those of GDP and inflation, the results are still significant at 90 percent confidence levels. However, the RBI shows the significant response to a shock in investment at 95 percent confidence level. An increase in investment then provides the key driving factors for real estate bubble in Thailand.

Next, we consider the responses of RBI to shocks in financial market variables. Figure 10 shows the results from the BVAR model. The results show that the RBI display a negative response

to a shock in the Minimum Loan Rate at 10 percent level of significance. However, the RBI shows a negative response to a shock in FPI, which contrasts the theory. Nonetheless, the sizes of responses are very small and insignificant. In the case of credit variable, an increase in the ratio of credit to GDP reinforces the RBI in Thailand. However, the impulse response function to credit to GDP ratio is insignificant at 5 and 10 percent level.

Finally, we compare the effects of monetary policy variables. We consider the conventional monetary policy tool, i.e., the 1-day RePurchase rate (RP), and the macro-prudential policy, i.e., the Loan to Value (LTV) measure. The results from Figure 11 show that the both RP rate and the inversed LTV ratio have the insignificant effect on the RBI. The minimal decrease in RBI is observed after a shock in RP and LTV.

Figure 9 Responses of RBI to shocks in macroeconomic variables

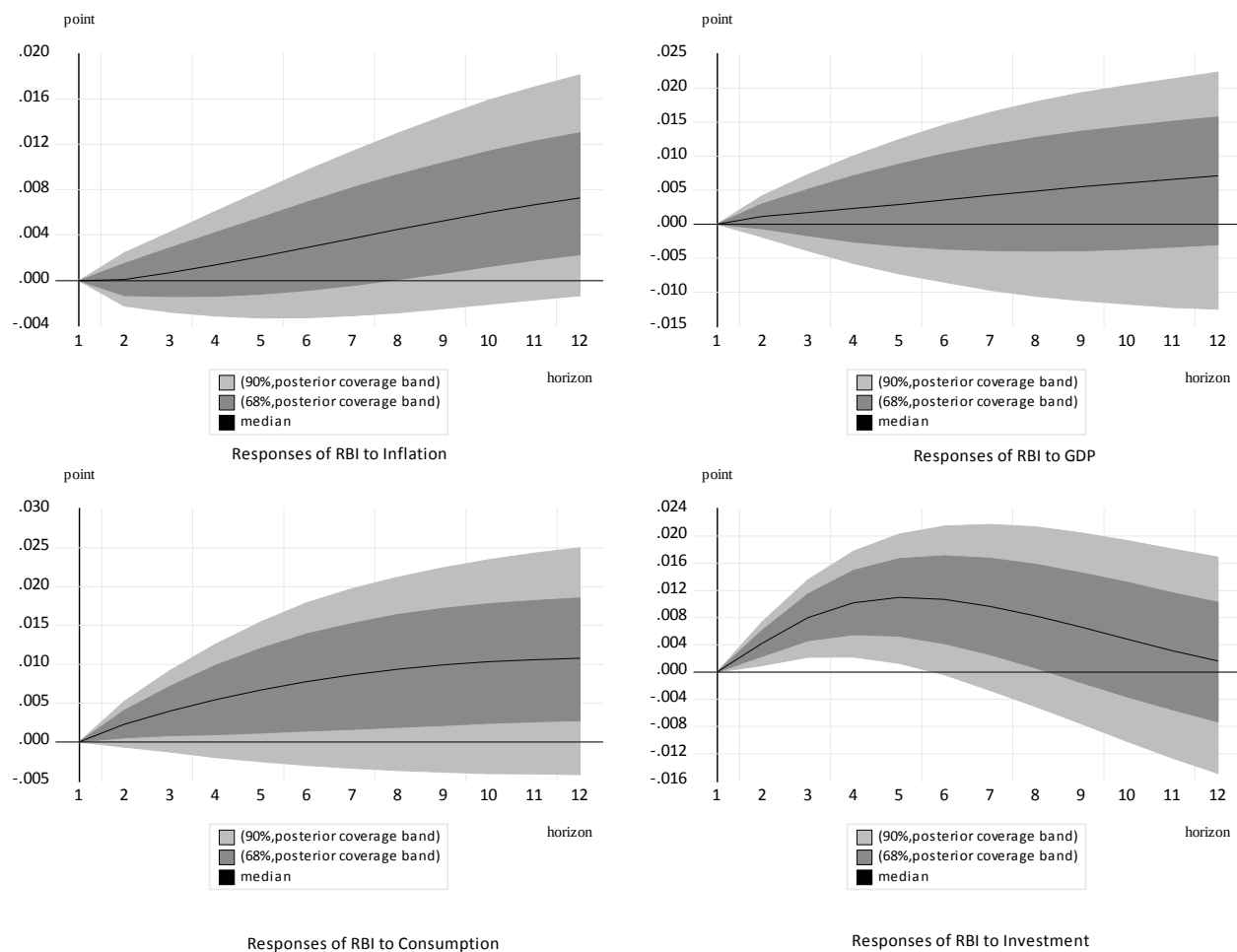


Figure 10 Responses of RBI to shocks in financial market variables

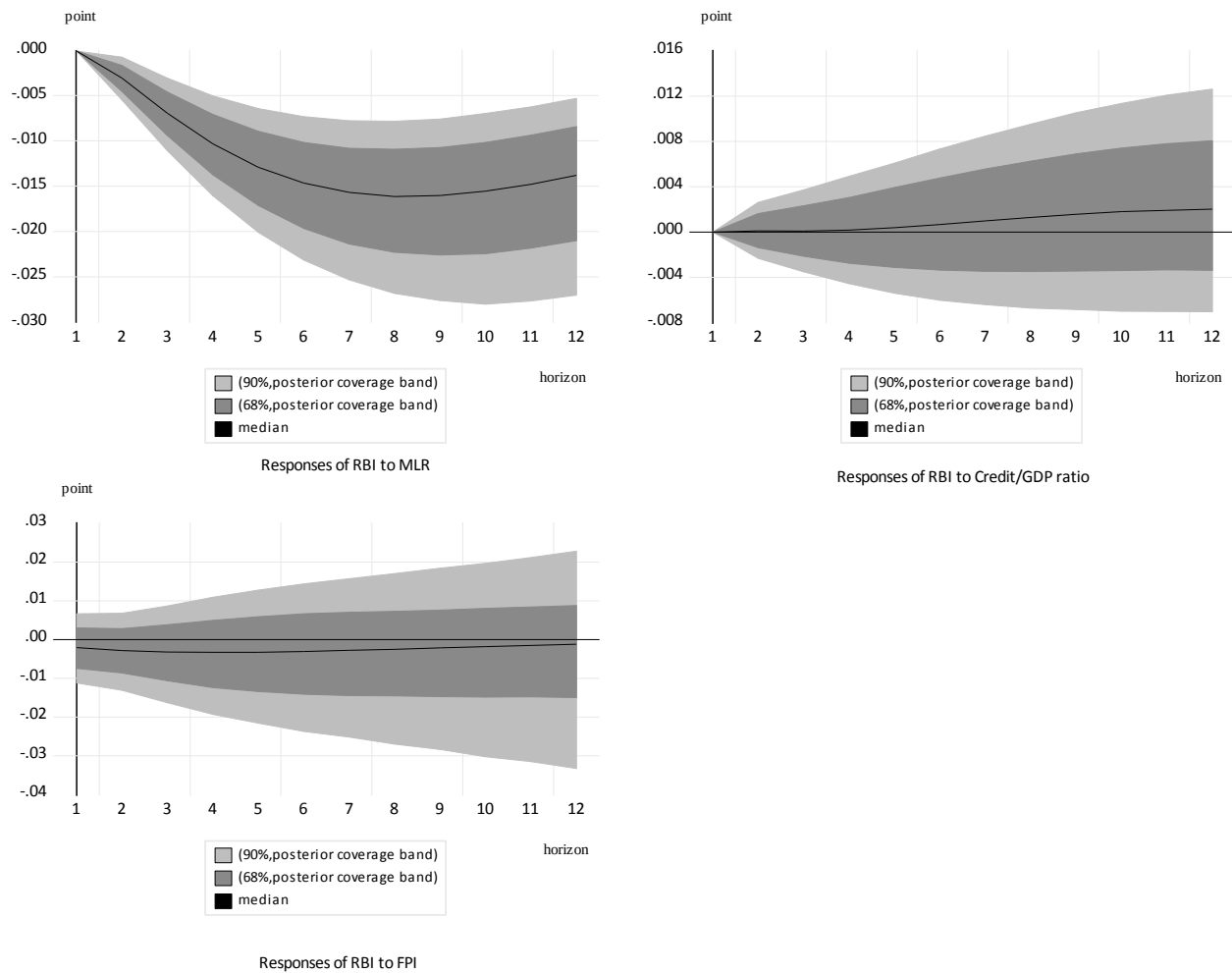
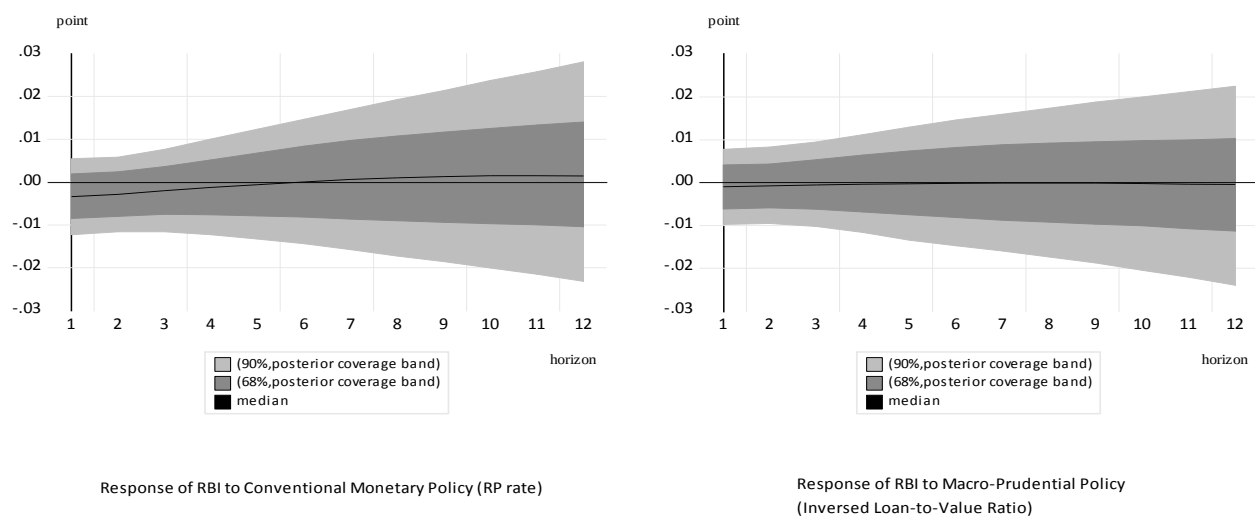


Figure 11 Responses of RBI to shocks in monetary policy variables



In sum, our empirical results provide the crucial implication for the factors causing bubbles in the real estate market. The key policy implication of the study can be concluded and presented in next chapter.

5. Policy Implications and Conclusion

5.1 Conclusion

This research investigates the real estate bubble in Thailand. We review the theory explaining the role of macroeconomics and financial market variables in driving the real estate bubbles. We propose the Real Estate Bubble Index (RBI) for Thailand. The methodology is based on the principal component analysis (PCA). We adopt the framework from the UBS based on availability of Thailand's data. The five sub-indices are used, i.e.

Z_1 : Indicators for the demand growth (Change in Town House Price index to Household Income)

Z_2 : Indicators for the price pressure (Town House Price index to Overall Consumer Price Index)

Z_3 : Indicator for financial sectors (Commercial Bank Personal Housing to nominal GDP)

Z_4 : Indicator for the production (supply side) in the macroeconomy (Construction plus real estate activities in relative to total output)

Z_5 : Indicator for the supply growth (Change in supply of real estate in relative to quantity household)

The results show our proposed real estate bubble index (RBI) from a PCA method. This series can explain the pattern of movement in real estate market in Thailand. Hence, we use the RBI as a proxy of real estate bubble in Thailand. The BVAR models are estimated. The results show that the investment, consumption and minimum loan rate are the important macroeconomic factors affecting the real estate bubble in Thailand. The credit to GDP ratio has insignificant effect to the movement of bubble. The total GDP has limited impacts on the RBI. Lastly, both conventional monetary policy (repurchase rate) and macro-prudential (loan-to-value ratio) tools have limited impacts on controlling bubbles.

5.2 Policy implications

The results of real estate bubble index and the BVAR on the responses of RBI to macroeconomic, financial markets and monetary policy variables provide the key implications for policy recommendation. We summarized the implications as follows.

1. The real estate bubbles could be the important cause of the macroeconomic imbalance in Thailand. The boom and bust in real estate bubbles are found before the Toom Yum Kung Crisis. Hence, monitoring the real estate bubble is the important part of the economic and financial stability. Recently, financial stability has been mentioned by Bank of Thailand as one of the key objective in the Monetary Policy Committee decision making. However, the focus of financial stability is based on household debt and non-performing loan in banks. The real estate bubble could be another proxy for financial stability in Thailand. Our proposed RBI could be applied for this purpose.
2. The recovery in housing market after crisis is slow. Hence, the stimulus policies are required to support the real estate sector after the crisis; for example, central bank asset purchasing plan, i.e. quantitative easing policy. Moreover, the asset purchasing plan could also be adopted, e.g. Mortgage-back security.
3. The private investment, minimum loan rate, inflation and consumption expenditure are the key factors relating real estate bubbles in Thailand. The recent slowdown of investment cycle in Thailand is one the main reason for current sluggishness in the real estate market, even though the interest rate is low. Moreover, an increasing in private investment expenditure could be the key warning sign of the future bubble in real estate market.
4. Monetary policy has insignificant effects on real estate bubbles. The tightening in the policy interest rate and LTV measure could not slow down the real estate bubble. Hence, additional policy tools are required to achieve financial stability.
5. The role of the loan-to-value (LTV) measure is still too early to evaluate. The data of LTV is available only after 2013. The further research may require evaluating the role of loan to value measure to real estate bubble in the future.

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