Thermodynamics Formulation of Economics

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A Thesis Submitted in Partial
Fulfillment of the Requirements for the Degree of
Master of Economics (Financial Economics)
School of Development Economics
National Institute of Development Administration
2019

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ABSTRACT

Title of Thesis	Thermodynamics Formulation of Economics
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Degree	Master of Economics (Financial Economics)
Year	2019

We consider a group of information-symmetric consumers with one type of commodity in an efficient market. The commodity is fixed asset and is non-disposable. We are interested in seeing if there is some connection of thermodynamics formulation to microeconomics. We follow Carathéodory approach which requires empirical existence of equation of state (EoS) and coordinates before performing maximization of variables. In investigating EoS of various system for constructing the economics EoS, unexpectedly new insights of thermodynamics are discovered. With definition of truly endogenous function, criteria rules and diagrams are proposed in identifying the status of EoS for an empirical equation. The rules and diagrams are named effect structure rules and the effect structure diagrams. With the rules, EoS are classified into two classes. Applying the conventional thermodynamics and the new rules, we can identify demand-side economics EoS. The price takes a role of intensive variable, demand quantity as extensive variable and average personal wealth possessed by consumers as temperature. The economics EoS is of the new class, the Class III. We proposed exact form of the economics EoS for econometrics analysis. At last we propose the way to perform Monte-Carlo simulation for the EoS. Having explicitly known EoS of the economy and its coordinates lays reasonable assumptions of Hamiltonian and partition function which are central aspects in application of statistical mechanics to economics and finance.

ACKNOWLEDGEMENTS

This work is supported by Graduate School of Development Economics, NIDA and Thailand Center of Excellence in Physics (ThEP Center). The author thanks the Institute for Fundamental Study, Naresuan University for research facility and Associate Professor Dr. Yuthana Sethapramote for discussion.

Burin Gumjudpai
January 2020

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CHAPTER 1

INTRODUCTION

1.1 Background and Significance of the Problem

Similarities of mathematical relations in economics and in physics have been noticed as well as mathematical analogies of the equation of motion of fluid dynamics to financial derivative pricing equation (Chatterjee, 2016; Jovanovic & Schinckus, 2017). Key features of science emphasizes on predictability power of empirical models to real phenomena. Repeatability of the prediction must hold under the same circumstances. Moreover science must be deducted from postulates awaiting confirmation coming from empirical facts or empirical laws, so-called the model. Considering epistemological approach, validation of what could be considered as science comprises of some criteria. In the past, economics and finance failed to passed epistemological sufficient conditions in the aspects that their theories could not be validated empirically. The recent decades have seen much improvement on theoretical validation from empirical facts in economics and finance. This is due to disruptive development of digital technology and data sciences (Chatterjee, 2016; Eichner, 1983; Schabas, 2009; Veblen, 1898).

When sociology and economics were still young, their mentality was not much different from physical sciences. Social sciences were coined social physics as their postulates and theories were necessarily laid out (Comte, 1856; Quetelet, 1869) and both deduction and induction (inference) reasoning were framed to the subjects. This is just as similar to how reasoning in physics does the role in its development. Tools and ideas were brought from old classical physics in developing social physics and some physics-trained people took some major parts in developing framework of economics in the early 20th century. For example, Irving Fisher (utility theory and monetary economics) and Jan Tinbergen (dynamics model of economics analysis). Ever since, social sciences and economics have been developing tools and concepts on their own rights. In finding resource allocation and price formation, economics develop its own concept up while physical approaches are to use statistical mechanics or physics of random walks or agent-based modeling. Apart from these subfields, there many other subjects of applied mathematics and physics such as fluid dynamics, complex networks, quantum mechanics, quantum field theory, path integrations, classical dynamics and classical thermodynamics that are under serious exploration in applying to economics, finance and social systems (Baaquie, 2007; Mantegna & Stanley, 1999; Richmond, Mimkes, & Hutzler, 2013). These approaches consider either individual agents either at microscopic scale or at each time scale in economy. Only classical thermodynamics views the problem aggregately and hence variables are evaluated at average. Gaps of these interdisciplinary knowledge are awaiting for researchers to explored.

1.1.1 Model Building Nature of Economics and Finance and Unification

Economics and finance are social science subjects with strong characters of model building. Both are also based on some postulates or assumptions which are empirically backedup with data and facts. Amongst social sciences, economics and finance inherit nature of scientific disciplines the most, due to their quantitative nature of study and their assumptions made as their first principles. Hence economics and finance could be serious scientific subjects such as physics or chemistry.

Unification of economics concepts and ideas into one single theory might not be important aim of economics due to complexities of the subject. The difficulty might be diversity or heterogeneities of human choices. The complication of choice might also come from technology development that makes human forms preference more and more complicated. Apart from that, human's memory (historical) effects that make microscopic agents of social system (human) much distinct from physical system such as particles, atoms or molecules. As a result, concepts and ideas in economics and social sciences are divided into schools of thoughts that can not be firmly practical empirical laws albeit the first principles (key assumptions) are well established, yet awaiting to be examined. On the other hand, in natural science, unification of theories into one single framework is key idea. Although there is no yet final unified theory of nature, nevertheless significant progress and advancements have been made in the past few decades.

It is learnt that financial crises in the past decades can not be predicted in frameworks of contemporary economics (Lux & Westerhoff, 2009). Unification of theoretical frameworks in economics and finance, in the same spirits as physics, may help leading direction of modifying theories and ideas in economics and finance, with even unnecessary of amending first principles of economics and finance. If there is a better economics theory, it should be more generalized and larger in coverage of the economical phenomena. It is far hoped that the unified framework is able to explain the larger economics and financial phenomena eventually. In the quest of unification of such theories, we shall begin with making analogies between social and physical systems as a first step. Along the way in future, we hope to follow track of physical theories unification.

1.1.2 Notices on Equilibrium State in Economics and Thermodynamics

We notice that both thermostatics (well-known as equilibrium thermodynamics) and economics are theories relying on existence of equilibrium states. Some variables of the two theories can be thought of the cause (or influence) and the effect. Moreover, similarities in mathematical formulations of both subjects are noticed by learners. Speculation that the two subjects have some similarities in their structures is not an insignificant issue. Historically, connections between the economics and thermodynamics had been abandoned and a formal links of the two subjects was raised long ago by Fisher and Samuelson (Fisher, 1892; Samuelson, 1960, 1990) albeit not turning to be neither a popular nor mainstream research discipline. Analogous theoretical frameworks of economics to thermodynamics would open new horizons and new perspectives of economics and financial concepts. This might improve the way we model economic system and the way we explain economic phenomena. Yet to come, further part of thermodynamics includes theory of phase transitions, if the true framework of thermodynamical economics is found, all techniques and theoretical frameworks of phase transitions would be as well much beneficial to its rich applications in economics and therefore to finance.

1.1.3 Mathematical Insuffciency of Equation of State in Thermodynamics

In elementary thermal physics, students are taught of the ideal gas law which is actually the equation of state of an ideal gas. The law states

$$PV = nRT, \tag{1.1}$$

whereas, P, V, T, n are pressure, volume, temperature and molar number of the gas and R is the gas constant. Before this we have learnt specific cases of this law. These are such as Boyle's law

$$V \propto \frac{1}{P}, \tag{1.2}$$

Amontons'law

$$P \propto T, \tag{1.3}$$

and Charles' law

$$V \propto T. \tag{1.4}$$

Each keeping all other conditions or variables constant. All these laws are relations of two variables. They are correct mathematically, however they are lacks of some physical sense. For example, look at Charles' law, $V = V(T) = \kappa T$, (κ is a proportional constant), the temperature can not directly make any changes in volume but instead the temperature makes change in pressure first and afterwards the pressure affects the change in volume. This gives a significant clue that the mathematics of the equation of state of an ideal gas $PV = nRT$ does not give a complete physical realization. It brings about our curiosity to investigate and check all other relation between these variables and all other types of equation of state in Thermodynamics.

1.1.4 Speculation of Missing Piece of Economics: the Equation of State

In modeling economics phenomena with econometrics, first consideration is to pick up a good assumption or suitable economics theory. Therefore we can consider some variables as independent variables and dependent variables and hence performing regression analysis on data. This is performed without the concept of equation of state, hereafter **the EoS**. The regressed equations are also constructed without considering of which variables have *exact differential* (state variables in thermodynamics or known as *stock* variables in economics) and of which variables have *inexact differential* (energy transfer in Thermodynamics or known as *flow* variables in economics).

In thermodynamics, there is a space of the theory. The coordinates of the space of theory form a set of manifold. Element of this set must be categorized as extensive coordinate (X), intensive coordinate (Y) and temperature (as a background quantity emerging from founding axiom) in the manifold,

$$\mathfrak{M} = \{X_i, Y_i, T\}, \text{ where } i = 1,2,3, \ldots . \qquad (1.5)$$

Note that in general some extensive variables in the subject are not necessary coordinates but *thermodynamics potentials* defined with the coordinates, for instance, internal energy ($U = U(X_i)$). In this manifold, all elements have roles of *thermodynamics coordinates*. All other variables are not included in the manifold. Within the elements of the manifold, there is a mathematical relation relating these coordinate variables. We are talking about the EoS. Having a space of the theory as stated, integration of variable with exact differentiality is independent of path in the coordinate space of the theory. On the other hand, integration of variable with inexact differentiality depends on path.

In economics, there is also manifold of the theory confirmed existence of the *value* function of commodities or other types of manifold in different contexts of mathematical analysis. However, the EoS aspects of the manifold's element members, stated above, are not considered. The distinction of which variables are to be coordinates or to be potentials, or of which variables are to be extensive or intensive, or of which variables are to be exact or inexact differential are not clearly made (Debreu, 1959; Mas-Colell, 1989). If there is such an EoS analogous concept in economics, a thermodynamically-analogous space of economics must be found together with the EoS which is indeed the equilibrium surface of the theory. Potential is to be found and the hope to see a complete analogous theory with fruitful application in economics is real intrigue to see in future. In the far future, we hope to have a new direction of relating different sets of measurements and hope to have some extension to standard economics theory.

1.2 Objectives

1) To improve mathematical expression of the equation of state in thermodynamics such that it can reflect deeper physical insights

2) To identify coordinates of the space for the equation of state of the thermodynamics formulation of economics

3) To formulate equation of state of an economics system

4) To introduce equation of state for econometrics modeling and Monte-Carlo simulation

1.3 Scope of the Study

1) Homogeneity of consumers, producers and commodities

2) Information spreads symmetrically and instantaneously throughout the market.

3) Market is effcient and is considered at clearing condition.

4) All variables are considered at equilibrium.

5) Taking the equation of state and natural thermodynamics space coordinates as a founding initial basis prior to deducing to other aspects

1.4 Impact of the Study

1) New investigation on thermodynamics formulation of economics

2) Possible new founding perspectives in econometrics modeling

1.5 Organization of the Thesis

The thesis is planned to lay out necessary understanding of thermodynamics and economics in chapter 2. We discuss previous and existing works on the area in chapter 3. The trial on constructing the EoS is reported in chapter 4 which was reported by the author in Gumjudpai (Gumjudpai, 2018). In chapter 5, there is improvement in mathematical expression of the equation of state in Thermodynamics and the author proposes new founding of effect structure diagram in the EoS with criteria rules for an EoS status of empirical or phenomenological formulae or relations. The finding in chapter 5 helps disproving the EoS proposed in chapter 4. In chapter 6, the EoS of group of consumers is proposed and discussed. We suggest EoS in modeling of demand function for further econometrics methodology and for Monte-Carlo simulation. Some results in chapter 5 and 6 are reported in Gumjudpai and Setthapramote (Gumjudpai & Setthapramote, 2019) and Gumjudpai and Setthapramote (Gumjudpai & Sethapramote, 2019). We conclude our thesis in chapter 7.

CHAPTER 2

THEORETICAL BACKGROUNDS

2.1 Essence of Physics

There are two branches of physics, the first are those derived from Hamilton's (least action) principle and the second is equilibrium thermodynamics (or thermostatics). We shall dub here these two branches as

1) **Mechanics** which includes physics derived from Hamilton's principle such as classical dynamics, electrodynamics, quantum mechanics, quantum field theory, general relativity and so forth and

2) **Equilibrium** thermodynamics (correct name should be thermostatics). Equilibrium thermodynamics is regarded as classical thermodynamics while non-equilibrium thermodynamics is named modern thermodynamics. Modern thermodynamics is the attempt to include the out-of-equilibrium state.

<u>Mechanics</u>: Key concept for mechanics is the *equation of motion* (EoM) which can be derived from Hamilton's principle. Paradigms of thermodynamics is without the Hamilton's principle hence is very different from the rest of physics. In mechanics, integrations of a postulated scalar quantity (the Lagrangian, L) over time, t or spacetime (in field theory) is the action

$$\mathcal{S} = \int L\,dt, \tag{2.1}$$

which is varied to obtain the Euler-Lagrange equation or Hamilton's equations which are the EoM of the theories. Lagrangian function defined as the result of kinetic energy subtracted by potential energy. Well-known EoM of classical dynamics is the Newtons second law of motion,

$$\sum F = ma. \tag{2.2}$$

Laws of physics are indeed a constraint surface embedded in some physical or abstract spaces. All the EoM with infinitely-given set of coordinate values are geometrically a constraint surface embedded in a space of the theory. The coordinates space of mechanical subjects are such as (x, y, z) (Euclidean space), $(q, \dot{q}, t)$ (generalized coordinate, generalized velocity, time) or (q, p, t) (generalized coordinate, canonical momentum, time).

Thermodynamics: Thermodynamics was formed with empirical formulae that serves as models of how substance behaves in relation with heat and temperature. These models are known as *equation of state* (EoS). The EoS is constructed not theoretically like the EoM but empirically. Chemists and engineers contributed greatly to the earlier developments of the subject before it was wrapped up by physicists in the early 20th century. These empirical formulae must be used in (plugged into) four postulate laws of thermodynamics in order to calculate physical quantities such as internal energy and energy transfer (i.e. heat and work). Considering simplest closed hydrostatics system, mechanical intensive coordinate (Y) is the minus sign of pressure $(-P)$, mechanical extensive coordinate X is volume (V), thermal intensive coordinate is temperature (T) and thermal extensive coordinate is entropy (S). Thermodynamics space is a 4-dimensional space (T, S, P, V) but we can view only 3 dimensions as (V, P, T), (V, P, S), (P, T, S) or (V, T, S) axes. Existing of infinite equilibrium states connected by an constant S process (isentropic or adiabatic) equivalently results in equilibrium surface Σ which is the EoS. Hence existence of the EoS results in two constraints:

1) S is fixed
2) there is a relation between (V, P, T).

EoS hence reduces 2 degrees of freedom out of the 4 degrees of freedom, resulting in a 2dimensional geometrical object described by a space of 3 coordinates (V, P, T). Therefore the EoS is a 2-dimensional constraint surface embedded in 3-dimensional thermodynamics space. The space enables one to define a thermodynamics potential function U as a function of coordinates, that is a *field*, of thermodynamics space. The constraint surface, Σ is in form of

$$g(X, Y, T) = 0, \tag{2.3}$$

whereas it reads

$$g(P, V, T) = 0, \tag{2.4}$$

for hydrostatics system. Work term, $\delta W = -P\mathrm{d}V$ is a combination of P and V, dubbed the *mechanical pair*. Heat term ($\delta Q = T\mathrm{d}S$) is of the same spirit of the mechanical pair. Postulates of thermodynamics define T (in the 0th law), internal energy U (in the 1st law) and S (in the 2nd law).

2.2 Thermodynamics

All thermodynamics quantities are all derived from EoS embedded in thermodynamics coordinate space. Hence to obtain the EoS and to identify the thermodynamics coordinates are the most important. Let us see the consequence of thermodynamics postulates that result in what quantities and state to exist and how they are related (Buchdahl, 2009).

2.2.1 Thermodynamics Space, Extensive and Intensive Coordinates

Quantities in thermodynamics can be viewed quantitatively as *extensive quantities* and *intensive quantities*. Extensive quantities depend on size (i.e. boundary) of the system of consideration whereas the intensive quantities are in opposite, that is it is size independent. Examples of extensive quantities are volume (V), internal energy (U), work (W), heat (Q) and entropy (S). Examples of intensive quantities are pressure (P), temperature (T) and other specific quantities of the extensive quantities, i.e. any extensive quantities can be made intensive by dividing them with molar number or mass. The above-mentioned statement is not much useful here in picturizing the frameworks of thermodynamics. To see the frameworks, we need to consider these variables as coordinates of thermodynamics space.

In thermodynamics we have *thermodynamics space* with *thermodynamics coordinates*. Considering aspects of size-dependency and origin of variable, we can categorize thermodynamics coordinates into six types.

1) **extensive coordinate from mechanical definition**, (X):

Examples are volume (V), magnetization ($\mathcal{M}$), electrical polarization ($\mathcal{P}$)

2) **intensive coordinate from mechanical definition**, $(Y\ )$:

Examples are (negative value of) pressure $(-P)$, magnetic field intensity (B_0), electric field intensity (E),

3) **entropy**, (S):

Entropy is extensive coordinate from classical thermodynamics postulate-the second law.

4) **temperature**, (T):

Temperature is intensive coordinate from classical thermodynamics postulate-the zeroth law.

5) **molar number or number of particles**, (n) or (N):

Molar number or number of particles are extensive coordinate for a system with mass exchanging into and out-of system.

6) **Chemical potential**, (μ):

Chemical potential is intensive coordinate for a system with mass exchanging into and out-of system. Chemical potential is viewed as a flow potential which is a thermodynamics coordinate, not thermodynamics potential.

A pair of coordinates from non-thermodynamics definition are mostly of mechanical or electromagnetic origins. Product of the pair forms work term, $\delta W = Y\,\mathrm{d}X$ which is a form of energy transfer-an entity as *flow* of some form of energy. The pair $(X, Y\ )$ is so-called *mechanical pair*. Examples are such as $-P\mathrm{d}V$ (mechanical work) or $B_0\mathrm{d}\mathcal{M}$ (paramagnetic work). The pair (S, T) forms heat term, $\delta Q = T\mathrm{d}S$. The pair (n, μ) forms a term describing energy that transferred with mass, μdn into or out-of an open system. A system describing with (X, Y) as $(V, -P)$ coordinate is known as hydrostatic system as mentioned before.

These thermodynamics coordinates are continuous function such that they are path independent hence their differentials are said to be *exact differential* where we use symbol df for exact differential of a function. On the other hand, energy transfer (of flow) term, i.e. heat (δQ), work (δW) and μdn are path-dependent hence their differentials are inexact differential where the symbol δf denotes inexact differential.

2.2.2 Thermodynamics Potentials and Legendre Transformation

Thermodynamics potentials are not the coordinates. They are path-independent hence their differential are exact. They are regarded as *state function* because they are function of thermodynamics coordinates. The status of thermodynamics potentials is similar to gravitational or electric potential field in mechanics or electromagnetism such that both of them are function defined in coordinate space. Examples of thermodynamics potentials are the internal energy (U) which is a sum of kinetic energy at all degrees of freedom of the system, enthalpy (H), Helmholtz free energy, $(\mathcal{F})$, Gibbs potential, (G) and grand potential, (Φ). One potential can be transformed to new potential with new physical implication. This transformation is Legendre transformation. The Helmholtz free energy is derived from *Legendre transformation* of total internal energy with dual coordinates (T, S). It is defined with the Legendre tranformation as internal energy subtracted by useless thermal energy (TS),

$$\mathcal{F} = U - TS, \tag{2.5}$$

hence $\mathcal{F}$ is the useful energy of the system. Gibbs potential, (G) is total useful work plus the energy that keep the system in its shape or its structure, PV

$$G = \mathcal{F} + PV = U - TS + PV. \tag{2.6}$$

If PV can be transformed to energy, it contributes also to useful energy. Hence G is larger than $\mathcal{F}$ and it is known as the *exergy* of a system.

2.2.3 Closed System and Isolated System

A *closed system* is the system forbidden any exchange of mass between the system and environment. It might allow form of energy transfer as heat flow and work receiving to the system or given from the system. An *isolated* (or *adiabatic*) system does not allow any exchange in mass or energy transfer between its internal and external regions.

2.2.4 Quasi-static Process and Types of Processes

A system in equilibrium when evolving, it goes out of equilibrium. Its coordinates changes to new values. It is said that the system has been through some process.

In out of equilibrium condition, value of coordinates cannot be measured or realized because they are

1) time dependent
2) space dependent

and most cases, the dependencies are not well-defined or known, i.e. they are indeterministic. In order to have some process, we have to infinitesimally changes the coordinates to new values and wait until it soon comes back to equilibrium state. The coordinate value at new state can be measured. We can interpolate the equilibrium path between the two states such that the curve of process can be realized. This type of process is the *quasi-static process*. The process with constant T is called *isothermal*. For constant S, it is called *isentropic*. For constant P, it is called *isobaric*. For constant V, it is called *isovolumic*. In the process without heat flow, $\delta Q = 0$, it is known as *adiabatic*. These terminologies will be referred to later in this thesis.

2.2.5 Kelvin-Clausius-Planck "Classical" Postulate Laws

Traditionally, postulates of thermodynamics taught in universities follows the foundation laid and brought together by Kelvin, Clausius and Planck. These postulates known as classical version state that

1) The Zeroth Law

The zeroth law defines thermal equilibrium between bodies. The thermal equilibrium condition is that there is no heat exchange between bodies. Thermal equilibrium implies that there is a single-value scalar function that can be ordinal ranked from low to high. This scalar is hence the temperature, T. Given temperature a postulated quantity by the zeroth law, at thermal equilibrium temperature must be time independent (steady state) and the same everywhere in the system (homogeneity).

2) The First Law

The first law gives existence of internal energy, U. The internal energy is sum of kinetic energy at all degrees of freedom, $E_{k,j}$, in the system. It is expressed as

$$\mathrm{d}U = \delta W + \delta Q + \mu \mathrm{d}n. \qquad (2.7)$$

The first law also introduces the connection between U to the changes in mechanical work, δW and heat, δQ where δ denotes inexact differential. The last term, $\mu \mathrm{d}n$ tells how much energy that amount of substance $\mathrm{d}n$ mole carries into or out-of the system. μ is the chemical potential. Moreover, this is how mechanics has a channel to link with thermodynamics. The law introduces thermodynamics space coordinates of (Y_i, X_i) (such as pressure and volume (P, V)) that define path-dependent quantity,

$$\delta W = \sum_i Y_i \mathrm{d}X_i, \qquad (2.8)$$

such as the hydrostatics work term, $\delta W = -PdV$.

3) The Second Law

This law introduces new postulated quantity describing degree of Disorder the entropy, S. The law tells that total entropy of the system and environment never decreases, i.e.

$$\mathrm{d}S_{tot} \geq 0, \qquad (2.9)$$

and for a closed system, entropy is maximized at thermodynamics equilibrium. The equality is the case of reversible process that keeps all steps of the process quasi-statics.

4) The Third Law

The third law states that it needs infinite steps of process to approach absolute zero temperature at which it is postulated to be zero entropy state.

5) Other Consequences of Laws of Thermodynamics

The first law introduces heat term δQ which is defined as consequence of the zeroth and the second law,

$$\delta Q = T\mathrm{d}S. \qquad (2.10)$$

This introduces thermodynamics space coordinates (T, S). For a system with n moles of substance, consequence of these three laws gives that,

$$U = U(S, X_i, n). \qquad (2.11)$$

Hence U is a potential function or field defined on the thermodynamics coordinate space of (S, V, n). Once U is established as extensive potential, for a scalar factor a as multiplication factor of the system, it follows that

$$aU(S, X_i, n) = U(aS, aX_i, an), \qquad (2.12)$$

and the *Euler's equation* can be directly derived,

$$U(S, X_i, n) = TS + \sum_i X_i Y_i + \mu n. \qquad (2.13)$$

The hydrostatic case, it reads

$$U(S, X_i, n) = TS - PV + \mu n. \qquad (2.14)$$

2.2.6 Equation of State - an Empirical Law

The postulates stated above do not relate the coordinates (Y_i, X_i, T, S, μ, n) together. In a closed hydrostatic system, the coordinate space are $(V, -P, T, S)$. If there is such a set of states with fixed value of S, the coordinate space are $(V, -P, T)$. However to have such a set of constant S naturally comes with a constraint relation obtained from observational or experimental facts. This is the EoS, $g(X, Y, T) = 0$, or in a closed hydrostatic system, $g(V, P, T) = 0$, that exists in the process of constant entropy (isentropic), that relates the three coordinates together. Familiar examples are the EoS of an ideal gas,

$$PV - nRT = 0, \qquad \text{(ideal gas law)} \qquad (2.15)$$

and of paramagnetic substance,

$$\mathcal{M} - \mathcal{C}\frac{B_0}{T} = 0, \qquad \text{(Curie's law)} \qquad (2.16)$$

where $\mathcal{C}$ is the Curie constant. The EoS is hence defined at *thermodynamics equilibrium*. Form of the relation is found empirically. In classical thermodynamics, the system must be identified with boundary of inner and outer regions. The mechanical and thermal coordinates must be found such that they describe the work and heat term. Empirical facts relate these coordinates together in the EoS - a constrained surface embedded in the space. With this setting, thermodynamics potential (such as U) is defined with these coordinates. There might be a composite physical system as such there are two mechanical pairs in one system. The system is known as a composite type however the constraints are separable as example, equation (2.15) and (2.16) when combining,

$$PV + \mathcal{M} - nRT - \mathcal{C}\frac{B_0}{T} = 0. \qquad (2.17)$$

This is realized as $g(X_1, X_2, Y_1, Y_2, T) = 0$ but we need to keep in mind that it is made from two independent EoS. Hence in composite physical system, there could be finite number of independent EoS,

$$g(X_i, Y_i, T) = 0, \qquad i = 1,2,3, ... \tag{2.18}$$

2.2.7 Thermodynamics Equilibrium Conditions

As consequence of these postulates, to acquire *thermodynamics equilibrium*, three physical conditions must be obeyed at the same time. These are

1) **Thermal equilibrium:**

Temperature takes the same value throughout the whole system and it must be time independent. The system has no thermal process at thermal equilibrium.

2) **Mechanical equilibrium:**

Y variables (such as P or B_0) take the same value throughout the whole system and it must be time independent. The system has no mechanical or other types of motion.

3) **Chemical equilibrium:**

μ variable takes the same value throughout the whole system and it must be time independent. At chemical equilibrium, there is no diffusion or exchange of any chemical components between different parts of the system.

Since thermodynamics equilibrium is kept along the process without any entropy production. Equilibrium condition between region A and region B of a system is hence the balance of each term in the below equation,

$$\mathrm{d}S = 0 = \left(\frac{1}{T_A} - \frac{1}{T_B}\right)\mathrm{d}U_A + \left(\frac{P_A}{T_A} - \frac{P_B}{T_B}\right)\mathrm{d}V_A - \left(\frac{\mu_A}{T_A} - \frac{\mu_B}{T_B}\right)\mathrm{d}n_A. \tag{2.19}$$

It is important to note that at thermodynamics equilibrium, S is maximum whereas all thermodynamics potentials, U, H, F, G, Φ are minimized.

2.3 Demand, Supply and Price

Following Adam Smith and the *classical school of economics*, market is self-regulating system and advocation of freely competition (Laissez Faire) should be supported so that wealth can be achieved the most. The self-mechanism is well known as the invisible hand. The classical school brought the theory of value to the concept of price at equilibrium (Debreu, 1959; Mas-Colell, 1989).

In economy, price, Pr_x of a good x, is defined at market equilibrium with equality of demand and supply quantities, (Q_x^d, Q_x^s). The demand quantity function could be function of price (Pr_x), price of the other related good y, (Pr_y) and income (Ic) (Luenberger, 1995; Mas-Colell, Whinston, & Green, 1995; Pindyck & Rubinfeld, 2013),

$$Q_x^d = Q_x^d\left(Pr_x, Pr_y, Ic\right). \qquad (2.20)$$

Supply quantity is function of price (Pr_x), price of other related good (Pr_y) and technology level $(Tech)$,

$$Q_x^s = Q_x^s\left(Pr_x, Pr_y, Tech\right). \qquad (2.21)$$

Point-price elasticities of demand and supply are the percentage change in quantity with respect to one percentage change in price,

$$E_x^d = \left(\frac{\Delta Q_x^d / Q_x^d}{\Delta Pr_x / Pr_x}\right) \quad \text{and} \quad E_x^s = \left(\frac{\Delta Q_x^s / Q_x^s}{\Delta Pr_x / Pr_x}\right). \qquad (2.22)$$

Both elasticities are characters of the goods and market in the view of consumers (buyers) and producers (sellers). That is for a market with some particular properties or characters of buyers and sellers, there are some particular function of the elasticity. It is clear that the price elasticities are not constant but functions of price and quantity of goods,

$$E_x^d = E_x^d(Pr_x, Q_x^d) \quad \text{and} \quad E_x^s = E_x^s(Pr_x, Q_x^s). \qquad (2.23)$$

There are other types of elasticities but are not required in the context of discussion in this thesis.

2.4 Deriving of Demand and Supply Functions

Neoclassical school of economics introduces a few assumptions to enlarge the scope of the classical school. The neoclassical school hypothesizes that individuals are rational and have *preferences* of consumption for the demand side and of production for the supply side. Hence this is not the case of Robinson Crusoe economy which assumes one consumer, one producer and two goods.

Preference associates with values. The preference is assumed to come with information symmetry and choices are made independently by taste or needs of each individual. Assumption of preferences and choices arises due to *scarcity* of resources. Neoclassical theory seeks for mechanism of understanding how scarce resources are *allocated* so that the price equilibrium is discovered. This mechanism should hold in describing behaviors of both consumers and producers. Hence *price formation* and *resource allocation* are key ideas of the neoclassical school. Consumers' behavior can be described with maximization of utility function,

$$\mathbb{U} = \mathbb{U}(Q_x, Q_y). \qquad (2.24)$$

Each consumer has his or her own satisfaction or needs of commodities so that the preference exists and the choices are made. Hence utility function is an individual aspect. In buying commodities, consumers want to obtain maximum utility. The condition max[$\mathbb{U}$] of the buyers therefore explains of consumer behavior. From top-down theoretical method, the demand function is deducted or derived from the maximization of utility function under budget constraint line, using indifferent curves which represent ordinal level of utility. The supply function is derived with the minimizing cost of production, $Cost$ and maximization of the profit,

$$\pi(Q_x) = Rv(Q_x) - Cost(Q_x), \qquad (2.25)$$

where Rv is revenue. Both optimizations employ the Lagrange multiplier method. On the empirical side, demand and supply functions can be constructed from econometrical method which helps building a close-to-realistic model of economy.

2.5 Theory versus Empirical Laws

Empirical laws are constructed from real-world observations or experiments. Albeit under some simplified assumption, empirical laws is about facts rather than about fundamental issues. Empirical laws can predict and forbid events or phenomena to happen. Empirical laws do not tell us why or how such things happen or can not happen. Theories, unlike empirical laws, are to explain facts or explain empirical laws that how or why things happen or can not happen. In thermodynamics, EoS is empirical law and is explained by physical theory. For example, the ideal gas law is explained with the kinetic theory of gases derived from classical dynamics. In economics, laws of demand and supply are empirical. Law of demand is explained with the consumer behavior theory requiring of maximization of utility. Law of supply is explained with production theory which requires maximization of profit. This is sense of serious scientific spirit in economics. We hope to find economics empirical law in the style of the EoS in thermodynamics which is to be explained with theories in traditional economics or in the new thermodynamics formulation of economics.

2.6 Physical Agents and Social Agents

System is what we consider and must possess some properties. A system does not need to have well-defined boundary. Complexities in modelling social system have its root in aspects of social agents. These are aspects to consider when modelling laws for physical or social systems or agents:

1) Names

Physical agent: a particle named electron

Social agent: a man named Burin

2) Properties (in physics) or Status Elasticities (in social sciences)

Physical agent: (electron) three properties are (1) negative charge, (2) spin and (3) mass. Properties are mathematical variables. Social agent: (Burin) status: (1) teacher (with teaching duty elasticity) (2) father (with father duty elasticity) (3) citizen (with citizen duty elasticity, e.g. voting, paying taxes) ...

3) Duties

Physical agent: (electron) with negative charge property, it repulses other negative charges and attracts positive charges, i.e. it obeys Coulomb's law and equation of motion. Other properties follow different laws. Social agent: (Burin) with his teaching duty elasticity, he must teach three hours a day and so on.

4) Choices or Options

There are no options (or rights) in physical systems to choose but options exists in social system, evoking game theory to take part in social sciences (e.g. in economics). For example, electron must behave as dictated by physical law and it has no rights. Teacher has rights to use staff parking slot but he might choose not to use it.

Since social agents have options to do or not to do, hence they can create *risk of defaulting in doing duties*. For example, it is possible that teacher does not teach as given by a rule relating his teaching duty elasticity. Financial risk of defaulting in paying debts also has its root from option making.

5) Uncertainties or Systematic Risks

For classical physics, system is deterministic and uncertainties (or physical risks) are negligible. For quantum mechanics, uncertainty is associated within the equation of motions. Systematic risk is also important concept in subject of finance, e.g. in portfolio theory as systematic risk is unavoidable by optimization of financial portfolio.

The forth aspect, option and risk created from option making, is hence main reasons why social systems is much more complicated than physical systems.

CHAPTER 3

LITERATURE REVIEW

We shall from now discuss previous works of thermodynamical approach to economics here. Not to be miss-leaded, this work is not about directly taking human economics activities as mechanical or thermal processes and later evaluating these activities in thermodynamics terms. This direction of viewing are indeed *thermodynamics of economy* rather than thermodynamics formulation of economics (Chen, 2005; Hammond & Winnett, 2009; Jenkins, 2005; Roegen, 1971; Tishin & Baklitskaya, 2008). It is not our scope of study to consider statistical behavior of economy agents in pricing of commodities as in contemporary financial physics (or fashionably-econophysics) or financial mathematics (or fashionably-financial engineering). In doing such, it replaces the concept of utility and all those follow from which.

3.1 Walras, Fisher and Samuelson

Thermodynamics and economics share similarities in that they consider subsystem in interaction and consider dual quantities that represent state of the system. One variable represents content of the system (extensive coordinate) and the other represent influence (or force) that the subsystems interact to each other (intensive coordinate). Price is similar to pressure that price is represent of level of willingness to trade. Both price and pressure are intensive.

L. Walras proposed the concept of utility (Walras, 1909). The balance of price, which is gradient of utility function with respect to quantity of good, determines equilibrium. This is similar concept as in classical mechanics. The balance of force as gradient of potential energy with respect to position coordinates. Hence utility is thought as potential energy in classical mechanics.

Looking at thermodynamics, position coordinates in classical mechanics are analogous to thermodynamics coordinates as J. Willard Gibbs (physicist) is a prime advocator. A student of Gibbs, I. Fisher hence propose to substitute the idea of economics-classical mechanics connection with the idea of economics-thermodynamics connection. Fisher kept the Walrasian idea of utility gradient and its equilibrium, but the differentiation in the gradient is with respect to thermodynamics coordinates. In this setting, utility, $\mathbb{U}$ is thought as internal energy, U in thermodynamics (Fisher, 1892).

Thermodynamics connection to economics turn to be of much less interest while the economics community witnessed profound application of mathematical analysis to economics (Debreu, 1959). The connection initiated by Walras and Fisher were gradually forgotten. Moreover, mismatching of potential energy minimization and utility maximization further disfavored connection trial. From this situation, although utility connection to potential energy or internal energy might be wrong but it does not imply that the connection is not possible. There are also other combinations or other ways of matching variables in thermodynamics to those variables in neoclassical economics. In the past century, connection between economics and thermodynamics formulation had always been suspected to exist. As P. A. Samuelson’s critique (Samuelson, 1960):

> *“The formal mathematical analogy between classical thermodynamics and mathematic economic systems has now been explored. This does not warrant the commonly met attempt to find more exact analogies of physical magnitudes - such as entropy or energy - in the economic realm. Why should there be laws like the first or second laws of thermodynamics holding in the economic realm? Why should ’utility’ be literally identified with entropy, energy, or anything else? Why should a failure to make such a successful identification lead anyone to overlook or deny the mathematical isomorphism that does exist between minimum systems that arise in different disciplines.”.*

3.2 Lisman (1949)

J. H. C. Lisman gives analogy of utility, $\mathbb{U}$ to entropy, S. Function of resource constraint is analogous to the conservation of internal energy, U. He proposed expenditure function $Pr\ \cdot Q$ as analogous to the product of pressure and volume $P\ \cdot V$ of the ideal gas EoS (Lisman, 1949). E. Smith and D. K. Foley argued in (Smith & Foley, 2008) that the dual analogous $Pr\ \cdot Q$ to $P\ \cdot V$ is wrong because $Pr\ \cdot Q$ is derived from Legendre transformation of their settings hence they claim that Pr and Q should not be a mechanical pair in the resemble ideal gas EoS. We disagree with Smith and Foley (Smith & Foley, 2008) in this point. We point out that there is general definition of mechanical pair (Y, X). The pair (P, V) is just one of infinite pairs in nature. It is not necessary to compare or analogy $Pr\ \cdot Q$ with (P, V) and not necessary to make analogy of economy to only ideal gas EoS. Instead one should consider $Pr\ \cdot Q$ as mechanical pair (Y, X) of a yet-to-be known form of EoS. Here in our thesis, we use (Pr, Q) as mechanical pair but not as resemble of ideal gas EoS. Our initial setting comes from effect structure of the EoS and Pr and Q fells into mechanical pair of the Class III of our EoS classification. Our effect structure criteria is new and has never been considered before.

3.3 Saslow (1999)

In the work by Saslow (Saslow, 1999), Marshall's concept of economic surplus in neoclassical economics is taken as initial point. Surplus Ψ is the difference between total utility, $\mathbb{U}$ and total wealth $\mathcal{W}$ of small country,

$$\Psi = \mathbb{U} - \mathcal{W}. \qquad (3.1)$$

The surplus is not negative. For least develop economy, $\Psi =\ 0$. Utility function, U is analogous to negative value of internal energy $-U$, resulting that buyer's total wealth $\mathcal{W}$ is analogous to the minus value of Helmholtz free energy, $-\mathcal{F}$. There is sign error in Saslow (Saslow, 1999) at the initial setting and it is corrected here as follow. Ψ is analogous to $-TS$, value of money λ to P, amount of money M to V , price Pr to chemical potential $-\mu$, and commodity quantity Q to molar number n. Maximization of $\mathbb{U}$ (or maximizing $-U$) is hence minimization of U and $\mathcal{F}$. In this view, wealth is conserved as $\mathcal{F}$ is conserved for reversible process.

We argue here why it is not appropriate to make analogies of price Pr to chemical potential $-\mu$ and commodity quantity Q to molar number n. Firstly there is no EoS for (μ, n) coordinate because the EoS is exists in closed and equilibrium condition only. Secondly, when considering matter exchange between internal sectors of the system, demand side and supply side should be considered separately. Chemical potential describes diffusion of matter inside system and it deals with only internal exchange process, hence if it is used for the analogy, it considers demand side and supply side together, not separately. Saslow (Saslow, 1999) does not say about the existence of an EoS.

3.4 Thermodynamics Analogies of Macroeconomics

This approach to macroeconomics was proposed by Mimkes (Mimkes, 2006). The idea begins from noticing that the first law of thermodynamics (2.7),

$$\mathrm{d}U = \delta W + \delta Q,$$

shares the similar mathematics as capital balance equation of macroeconomics,

$$\mathrm{d}(GDP) = \delta(\mathbb{C}) + \delta(\mathbb{I}), \qquad (3.2)$$

where $\mathbb{C}$ and $\mathbb{I}$ are consumption and investment (assuming no government spending and closed system). Both are expressed in first-order differential forms and with this initial setting, analogous thermodynamical macroeconomics formulation is deducted. A macroeconomic system inevitably relies on the concept of entropy and indeed macroeconomics system is driven by entropy (Rosser, 2016). This work does not consider any aspect on the EoS either. This thesis considered only microeconomics aspects and we mention this work for the sake of further interest.

3.5 Constrained Optimization and General Equilibrium

3.5.1 Thermodynamics View of General Equilibrium, Constraint Conservation and Preference

In the year 2007 to 2008, fine analyses of connection between classical thermodynamics to neoclassical thermodynamics was shown by Smith and Foley (Smith & Foley, 2008) and Santos Costa e. Sousa (Sousa, 2007). These analyses proved that there is a mathematical isomorphism between both subjects.

It seems that neoclassical economics is mathematically mature and that it does not matter whether there is any analogies to thermodynamics. However there might be some additional insights or new significant results to obtain from considering the analogies or any connection and might give new contribution to economics and econometrics. It is quoted from page 9 of Smith and Foley (Smith & Foley, 2008),

"Economics, because it does not recognize an equation of state or define prices intrinsically in terms of equilibrium, lacks the close relation between measurement and theory physical thermodynamics enjoys.".

This emphasizes the need of EoS in formulating thermodynamics version of economics. It is suggested that similarities of economics and thermodynamics are noticed with three aspects:

1) Equilibrium determined by a set of pair of dual state variables

2) Constraint of conservation of a quantity in a closed system (not necessary isolated)

3) Preference of a quantity not to decrease in approaching equilibrium state.

Individual is regarded as an agent in economics system. Formal equivalence of utility function and its maximization for each consumer to the entropy function and its maximization are proposed and the second law of thermodynamics version of economics is what follows. The mechanical pair are bundles of quantity of commodities (extensive coordinates, Q) and prices of these commodities (intensive coordinate, Pr). Following marginalism of neoclassical economics, marginal rate of substitution (MRS) takes the role of temperature. When two agents have chance to trade if their MRS are equal, their is no trade. The first law of thermodynamics is the conservation of commodities (instead of the internal energy) and the second law is the maximization of utility as stated above. The EoS is suggested to relate three variables in a function, $g(Q.Pr, \mathrm{MRS}) = 0$ (Smith & Foley, 2008).

3.5.2 Constrained Optimization Problem

Mathematical foundation of neoclassical microeconomics and equilibrium thermodynamics can be shown to be isomorphism to each other in the perspective of constrained optimization (Smith & Foley, 2008; Sousa, 2007). As one considers behaviors of a system that evolves to maximize a function $y = y(x_1, \dots, x_n)$ under set of constraint conditions, $g_z = 0$ where $z = 1, \dots, m$. To be generalized, the constraints are also function of some value of these variables, x_1^0 obeying the same constraints and some other parameters, $k_j,\ j = 1, \dots, l$. The function y is maximized under the constraints g_z that is

$$\max_{x_1,\dots,x_n} y(x_1, \dots, x_n) \text{ for } g_z(x_1, \dots, x_n, x_1^0, \dots, x_n^0, k_1, \dots, k_l) = 0 \colon z = 1, \dots, m. \qquad (3.3)$$

In maximizing of this function, Lagrangian, $\mathcal{L}$ of the problem is

$$\mathcal{L}(x_1, \dots, x_n; k_1, \dots, k_l; \lambda_1, \dots, \lambda_m)$$
$$= y(x_1, \dots, x_n) + \sum_{z=1}^{m} \lambda_z g_z(x_1, \dots, x_n; x_1^0, \dots, x_n^0; k_1, \dots, k_l). \qquad (3.4)$$

The maximized objective function y can be obtained with Lagrange multipliers method. This is to solve $m + n$ equations:

$$\frac{\partial y}{\partial x_i} = -\sum_{z=1}^{n} \lambda_z \frac{\partial g_z}{\partial x_i} \qquad i = 1, \dots, n\,, \qquad (3.5)$$

$$g_z = 0 \qquad z = 1, \dots, m\,, \qquad (3.6)$$

which are equivalent to

$$\frac{\partial \mathcal{L}}{\partial x_i} = 0 \qquad i = 1, \dots, n\,, \qquad (3.7)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_z} = 0 \qquad z = 1, \dots, m\,. \qquad (3.8)$$

For real-value differentiable functions y and g_z the maximum value exists if $m < n$ and gradient of all g_z are linearly independent. Optimal values of x_i and k_j exist,

$$x_i{}^* = x_i(k_1, \dots, k_l, x_1^0, \dots, x_n^0). \qquad (3.9)$$

Hence there exist optimal values of y which is denoted with y^*,

$$y^* = y^*(k_1, \dots, k_l, x_1^0, \dots, x_n^0). \tag{3.10}$$

The function y is maximized to give optimal value y^* in a process when being reached equilibrium.

3.5.3 Constrained Optimization in Thermodynamics System

The problem is about maximizing of entropy S_1 and S_2 of composite systems - in this case supposing two subsystems, hence $j = 1{,}2$ Our function y here is

$$S = S(U, V, n). \tag{3.11}$$

Hence the maximization is of the entropy:

$$\max_{U_1, U_2, V_1, V_2, n_1, n_2} S = S_1(U_1, V_1, n_1) + S_2(U_2, V_2, n_2), \tag{3.12}$$

with constraints:

$$U_1 + U_2 = U_1^0 + U_2^0 \Longrightarrow g_1\left(U_1, U_2, U_1^0, U_2^0\right) = 0, \tag{3.13}$$

$$V_1 + V_2 = V_1^0 + V_2^0 \Longrightarrow g_2\left(V_1, V_2, V_1^0, V_2^0\right) = 0, \tag{3.14}$$

$$n_1 + n_2 = n_1^0 + n_2^0 \Longrightarrow \mathrm{g}_3\left(\mathrm{n}_1, \mathrm{n}_2, \mathrm{n}_1^0, \mathrm{n}_2^0\right) = 0. \tag{3.15}$$

Other variables are derived from this maximum as

$$\frac{\partial S_j}{\partial U_j} = \frac{1}{T_j}, \qquad \frac{\partial S_j}{\partial V_j} = \frac{P_j}{T_j}, \qquad \frac{\partial S_j}{\partial n_j} = -\frac{\mu_j}{T_j}. \tag{3.16}$$

The equilibrium conditions obtained agree with equation (2.19) and they are

$$\frac{1}{T_1} = \frac{1}{T_2}, \tag{3.17}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}, \tag{3.18}$$

$$\frac{\mu_1}{T_1} = \frac{\mu_2}{T_2}. \tag{3.19}$$

The optimal variables, x_i in this case are

$$U_j^* = U_j^*(U_1^0, U_2^0, V_1^0, V_2^0, n_1^0, n_2^0), \tag{3.20}$$

$$V_j^* = V_j^*(U_1^0, U_2^0, V_1^0, V_2^0, n_1^0, n_2^0), \tag{3.21}$$

$$n_j^* = n_j^*(U_1^0, U_2^0, V_1^0, V_2^0, n_1^0, n_2^0), \tag{3.22}$$

(where $j = 1, \dots, n$) such that the optimal function of entropy is

$$S^* = S^*(U_1^0, U_2^0, V_1^0, V_2^0, n_1^0, n_2^0). \tag{3.23}$$

A process on a path that entropy remains constant at S^* at all time is the reversible process.

3.5.4 Constrained Optimization in Consumer System

For n types of commodity, with consumed quantities, x_i where $i = 1, \dots, n$ (previously denoted as Q_s) utility of a customer is maximized,

$$\max_{x_1,\dots,x_n} \mathbb{U} = \mathbb{U}(x_1, \dots, x_n), \tag{3.24}$$

with budget constraint:

$$Pr_1 \cdot x_1 + Pr_2 \cdot x_2 + \cdots + Pr_n \cdot x_n \; = \; Pr_1 \cdot x_1^0 + Pr_2 \cdot x_2^0 + \cdots + Pr_n \cdot x_n^0, \tag{3.25}$$

which is

$$g_1(Pr_1, \dots, Pr_n; x_1^0, \dots, x_n^0; x_1, \dots, x_n) = 0. \tag{3.26}$$

Differentiation of $\mathbb{U}$ with respect to the coordinate variables, x_i giving marginal utility,

$$\frac{\partial \mathbb{U}}{\partial x_i} = \mathrm{MU}_i. \tag{3.27}$$

The equilibrium conditions are analogous to equations (3.17), (3.18), (3.19). It is

$$\frac{\mathrm{MU}_1}{Pr_1} = \frac{\mathrm{MU}_2}{Pr_2} = \cdots = \frac{\mathrm{MU}_n}{Pr_n}. \tag{3.28}$$

The optimal variables, x_i in this case are

$$x_j^* = x_j^*(Pr_1, \dots, Pr_n; x_1^0, \dots, x_n^0), \tag{3.29}$$

where $j \; = \; 1, \dots, n$. Optimal function of utility is

$$\mathbb{U}^* = \mathbb{U}^*(Pr_1, \dots, Pr_n; x_1^0, \dots, x_n^0), \tag{3.30}$$

A process on a path that utility remains constant at $\mathbb{U}^*$ at all time is the reversible process.

3.5.5 Constraint Changing Considered as Exogenous Effect

If constraints of the system are changed, for examples, the total internal energy changes or the budget of buying changes, the system moves from the equilibrium point to other point in such the way that S (in thermodynamics) or $\mathbb{U}$ (in neoclassical economics) is maximized. When $\mathbb{U}$ is at maximum, consumer receives additional endowment.

In economics, the constraint is the budget. Changing of budget is changing of constraint. If the consumer's budget comes from income of the consumer, we can consider the income as exogenous effect. We might pre-hypothesize that exogenous effect would influence the system via constraints. In this case the constraint is the budget constraint.

3.5.6 Existing of Intensive State Variables and Integrability

As consumed quantity, x_i is extensive variable, for an intensive state variable,

$$\pi_i = \pi_i(x_1, \dots, x_n), \tag{3.31}$$

to exist, 1) it must satisfy Maxwell's relations and 2) The state function obeys the optimized behavior of the system. These two conditions are equivalent to symmetric and negative semidefinite properties of Hessian matrix of y (Candeal, De Miguel, Induráin, & Mehta, 2001). In economics, this is

$$Pr_i = Pr_i(x_i, \dots, x_n), \tag{3.32}$$

which can be deduced from observations or experiments. This expenditure function can be built knowing all Walrasian demand functions. The Maxwell's relation is known in economics as Antonelli's integratibility condition and the second condition is known as economic integratibility (Sousa, 2007). It is noted that equation (3.31) is not an EoS in thermodynamics definition. Although it was understood as EoS according to Santos Costa e. Sousa(Sousa, 2007). In thermodynamics concept, an EoS must relate extensive variable,X, intensive variable, Y and temperature (also intensive) together as $f(X, Y, T) = 0$ which is not the same as equation (3.31). Proposal of Smith and Foley (Smith & Foley, 2008) hence introduced new concept of EoS into economics. In this thesis, our goal is, as well, to find the EoS, but with our novel concept of effect structure and Carathéodory approach that helps guiding us to express the EoS from empirical definition of process and to express the variable to be maximized.

3.6 Carathéodory Approach

3.6.1 Empirical Definition of a Thermal Process

Traditional approach of learning and developing of thermodynamics (the Clausius-Kelvin-Planck postulate laws of thermodynamics) is convenient in introducing basic ideas of thermodynamics. However the traditional (classical) approach fails to follow logical reasoning. These incomplete aspects in its foundation are (Münster, 1970):

1) Temperature and heat are necessarily postulated.
2) No distinction between empirical or physical entities to purely postulated or mathematical entities.

The work of Carathéodory, this is not the Carathéodory's theorem for the convex hull in convex geometry as discussed in page 27 of Mas-Colell (Mas-Colell, 1989), gives a strictly logical concept of thermodynamics based on only entities and concepts from empirical variables, e.g. mechanical pairs $(X, Y\ )$ and work term δW (Buchdahl, 2009; Giles, 1964; Landé, 1926; Münster, 1970).

In a homogenous system, extensive X variable is uniquely determined by intensive Y variable. The first empirical fact is

Theorem based on experience: "Processes exist leading to a change in X at constant or non-constant Y . The processes shall be dubbed *thermal processes*.".

The theorem says that we need a *mechanical pair* in defining a state of the system in XY plane. To recapitulate, in every neighborhood of any arbitrary initial state A of a physical system, there exists neighboring states that are not accessible from the state A along quasi-static adiabatic process.

3.6.2 Empirical Definition of Entropy

There is a quasi-static change in a homogenous isolated system from a state (X_0, Y_0) to (X, Y), i.e. it possesses *adiabatic accessibility*. The process is not a thermal process but an *adiabatic process*. Family of adiabatic processes constitutes a single-parameter family of curves covers completely a region in XY plane. One and only one adiabatic process passes through each point (X,Y) in the plane. Hence

$$s = s(X, Y). \qquad (3.33)$$

Example is the well-known adiabatic process or an ideal gas which is written as, $s(V, P) = s = PV^{\gamma}$. If there are two states of the system with only two adiabatic processes passing through them,

$$s'(X', Y') = s' \qquad \text{and} \qquad s''(X'', Y'') = s''. \qquad (3.34)$$

These two points are of two phases separated by an adiabatic wall. The two points (made of a mechanical pair) exist separately in equilibrium. If the wall is diathermic (allow heat flow), the two points can not co-exist. To recapitulate, in every neighborhood of any arbitrary initial state A of a physical system, there exists neighboring states accessible from the state A along quasi-static adiabatic process. The function s of each state gives definition of entropy function. For example, in reversible process of an ideal gas, entropy is constant hence

$$s(V, P) = PV^{\gamma} = P_0 V_0^{\gamma} = s(V_0, P_0), \qquad (3.35)$$

as well-known in text books.

3.6.3 Empirical Definition of Thermal Equilibrium and Temperature

Theorem of experience, and definition: "If two systems are adiabatically isolated from environment but diathermally, means allowing heat exchange, contact to each other, they are in *thermal equilibrium* only if the equation,

$$\mathrm{F}(\mathrm{X}', \mathrm{Y}' : \mathrm{X}'', \mathrm{Y}'') = 0, \qquad (3.36)$$

is obeyed. The condition can be attained only by a thermal process."

If there is the third system or the third phase in co-existing with the two system, a new theorem of experience is required,

Theorem of experience, and definition: "If the $'$ and $''$ systems are in thermal equilibrium to each other and the $''$ and $'''$ systems are in thermal equilibrium to each other, hence the $'$ and $'''$ systems must be in thermal equilibrium to each other. Each system must have measurable property (with thermometric properties, (X, Y)), which is $t(X, Y)$ such that

$$t'(X', Y') = t''(X'', Y''), \quad t''(X'', Y'') = t'''(X''', Y''') \quad \text{and} \quad t'(X', Y') = t'''(X''', Y'''). \qquad (3.37)$$

The property t is the *empirical temperature*. Temperature scale is expressed if we can define the function

$$t = t(X, Y), \tag{3.38}$$

with two reference points or with a reference point and a size of scale. The function $t = t(X, Y)$ is called the *thermal equation of state*.

For examples, for a hydrostatic system, the thermal equation of state is $PV = f(t)$ and for a paramagnetics system it is $B_0\mathcal{M}^{-1} = f(t)$. The thermal equation of state $t = t(X, Y)$ represents another family of single-parameter curves in XY plane, the family of *isothermal processes*.

3.6.4 Strictly Statement of the First Law

The above empirical axioms have given us a solid foundation of the zeroth law and the existence of entropy, S in the second law of "classical" postulates. According to Carathéodory, the first law must be defined with the mechanical pair, i.e. defined with purely empirical concept. Hence the definition of internal energy and heat are given empirically.

Theorem of experience, and definition: "The same amount of *work*, W is necessary to change a system from initial state to final state by adiabatic process. The amount of work is independent of whether or not the system is in equilibrium. The work that change the system is equal to the change in *internal energy*, U of the system. Hence

$$U_2 - U_1 = \int_1^2 \delta W, \tag{3.39}$$

in adiabatic process."

If the system evolves along some non-adiabatic path hence it is necessary to have new definition,

Definition: "For any process from initial state to final state, difference between the increasing (decreasing) in internal energy and work done into (out of) the system is called the heat, Q absorbed into (or exhausted out of) the system. That is

$$\int_1^2 \delta Q = (U_2 - U_1) - \int_1^2 \delta W \qquad \text{or} \qquad \mathrm{d}U = \delta W + \delta Q, \tag{3.40}$$

for any processes."

Hence the first law is the equation (3.39)for adiabatic process. For any process, the first law is the equation (3.40).

Since we have $s = s(X, Y)$ as in equation (3.33) for each adiabatic process hence s identifies entropy at empirical variables X, Y. From here, all classical treatment of thermodynamics have strong foundation given by Carathéodory empirical axioms so that all traditional thermodynamics theoretical consequences base on and can derived from the Carathéodory axioms.

3.6.5 Application to Other Types of System

The Carathéodory axioms allows any systems with extensive and intensive coordinates as a mechanical pair to form a first law if there are such states and processes that are analogous of thermodynamics states and processes. Following the axioms and considering other types of system, if one can empirically defined a mechanical pair and all analogous steps of the axiomatic reasoning, thermodynamics analogy of the system can be done. The steps of thought shall follow:

1) Having a thermal process which is equivalent to having a mechanical pair (X, Y) as natural coordinates of the theory's manifold

2) Adiabatic accessibility of state means having an adiabatic process which in turn defines *entropic state* $s = s(X, Y)$, hence, implying definition of entropy $S \equiv s(X, Y)$ here.

3) Thermal equilibrium is defined as $F(X', Y' : X'', Y') = 0$.

4) *thermal state* $t = t(X, Y) \equiv T$. It equivalent to

$$g(X, Y, T) = t - t(X, Y) = 0, \qquad (3.41)$$

hence EoS is to be found as a constraint surface Σ in the coordinate space of (X, Y, T).

5) Dimensional unit of $Y\mathrm{d}X$ is unit of δW, hence the unit of U. The first law is to be written. Multiplication of $T\mathrm{d}S$ should have the unit of $\mathrm{d}W$ and U. Conservation of U is to be checked.

The concept emphasizes on existence of sets of s and t infinite states that obey these conditions. This equally implies emphasizing on finding the right coordinates to form mechanical pair (X, Y). Definition of temperature as a function of the mechanical pair is indeed the EoS. Thermodynamics potential is defined on surface Σ - the EoS.

The first law follows from expression of δW as $Y\,\mathrm{d}X$. In this scenario, deriving EoS of a system is hence the key point.

This approach is different from previous works on thermodynamics analogous of economics in which the initial step is about looking for variable that is maximized (in analogy to entropy). We suspect here that the one-to-one correspondence might not be possible. The reason might be that physics is richer than economics in its fundamental quantities. These come with units of length, time, temperature, molar number (or number of particle), charge, mass, magnetic field. In economics, we have units of money, of quantity of goods, number of people (buyers or sellers) and maybe unit of utility? This is to be considered later in the thesis.

3.7 Direction of this thesis

In this thesis, we consider analogies between thermodynamics and neoclassical economics on the grounding that there is mathematical isomorphism between the two disciplines as shown by Smith and Foley (Smith & Foley, 2008) and T. A. Santos Costa e. Sousa (Sousa, 2007). Unlike all other previous works, our approach in this thesis clearly distinguishes variables of what are *thermodynamics coordinates* and what are *thermodynamics (field) potentials*. We emphasize on character of variables if they are *intensive* or *extensive* coordinates. We check which variables behave as *influence* and which variables are *passive* and can not influent other variables. Structures of the various EoS are characterized with our novel proposal of *effect structure*. The EoS are classified. This can be done with graphical diagrams of effect structure of variables of the EoS. Having known that the utility must be maximized, therefore it might play similar role to entropy maximization in thermodynamics, but yet to be checked.

Our approach follows Carathéodory axiom and, in addition we construct the EoS by observing naturalness of the EoS from economic assumption. Then we use effect structure analysis with the EoS and we try to identify which class the economics EoS falls into. Afterwards, economics' version of thermodynamical EoS is built. It is noted that this thesis takes no stand on how to predict more accurately real economic phenomena but to build first principle framework for what to deduce and to modify further to more realistic predictability.

CHAPTER 4

EOS OF A MARKET: A WRONG TRIAL

At a rough glance, in microeconomics system of a market, the equilibrium price is formed by equality of demand and supply quantities. This is very similar to how equilibrium temperature is achieved in thermodynamics. Temperature and price seem to share some similarity in its definition. Temperature is a function of internal energy and other variables such as pressure or volume, however it can not be measured directly. We need to measure it via other *thermometric properties* such as pressure, volume, electrical potential, electrical resistivity or spectrum of radiation. The zeroth law of classical thermodynamics, which is the founding postulate of the subject, proposes thermal equilibrium state to exist. This directly implies the existence of temperature even without its directly measurable nature. Price seems to be very similar. Value of commodities exists due to some assumption and price is formed by agreement in the transaction between sellers and buyers. This motivates trial work in this chapter on the equation of state of the market (Gumjudpai, 2018) in which whole market of small economy is a system and equilibrium price can be obtained as how temperature is reached at thermal equilibrium. Specific demand quantity (total demand quantity per buyer or per household) and total supply quantity are considered as intensive and extensive quantities. Traditional thermodynamics with four postulate laws are taken as first principles and EoS should come along from empirical fact. Alternative derivation to thermodynamics was theorized formally by Carathéodory whose theorems suggesting that EoS should exists as the beginning of reasoning, as a consequence all potentials exist on the EoS surface (Buchdahl, 2009; Giles, 1964; Münster, 1970). The existence of the EoS is the first key target as suggested by Carathéodory. We shall look at the building blocks of the EoS and afterwards constructing the EoS.

The work in Gumjudpai (Gumjudpai, 2018) is an initial trial of the author and it turns to be wrong. For a sake of learning from mistake, this chapter explains how.

4.1 First Trial on Setting the Zeroth Law of Thermodynamics Version of Economics

We assume there exists a 2-dimensional Σ surface for a system which is here a market, taking into account that three coordinates are quantities of demand (Q^d), supply (Q^s) and price (Pr). Price and temperature are both fundamentally postulated to exist before constructing theories of the subjects. Price is defined from market equilibrium (a condition when there is no excessive demand or excessive supply) as such the temperature is defined from thermal equilibrium. The zeroth law of thermodynamical paradigm of microeconomics should lay foundation of the concepts of market equilibrium and of price.

The mechanism of thermal and market equilibriums are quite different. At thermal equilibrium, the heat flow stops or the rate of in-flow and out-flow are equal. If the system is enclosed with heat insulating surface, energy inside the system is constant and the system stays still. At price equilibrium, the buying of commodity does not stop but still continue with the constant equilibrium quantity of bought product. We notice here and if we pursue with this initial setup then according to classical thermodynamics, we may state

"For three efficient, perfect competitive and clearing markets A, B, C in which there are trading of the same and homogenous normal goods. If A is in equilibrium with B and B is in equilibrium with C, hence A is in equilibrium with C. The equilibrium gives rise to existence of price function which can be ranked from lower to higher and is the character of market that determines if the market is in equilibrium with others. This type of equilibrium is the price equilibrium.".

There are other types of economics equilibrium such as interest rate equilibrium and exchange rate equilibrium.

4.2 Market with Q^d and Q^s Linear with Price

In a market, if the demand curve is of a function (Mas-Colell et al., 1995; Pindyck & Rubinfeld, 2013)

$$Q^d = k_s(Q^s) \cdot Pr + Q^{d,0}, \tag{4.1}$$

as in **Figure 4.1** (a). The proportional constant is $k_s(Q^s) < 0$ and the vertical intercept is $Q^{d,0}$. It could be elastic, unitary or inelastic demand depending on which section of the curve we consider. The supply curve is

$$Q^s = k_d(Q^d) \cdot Pr, \tag{4.2}$$

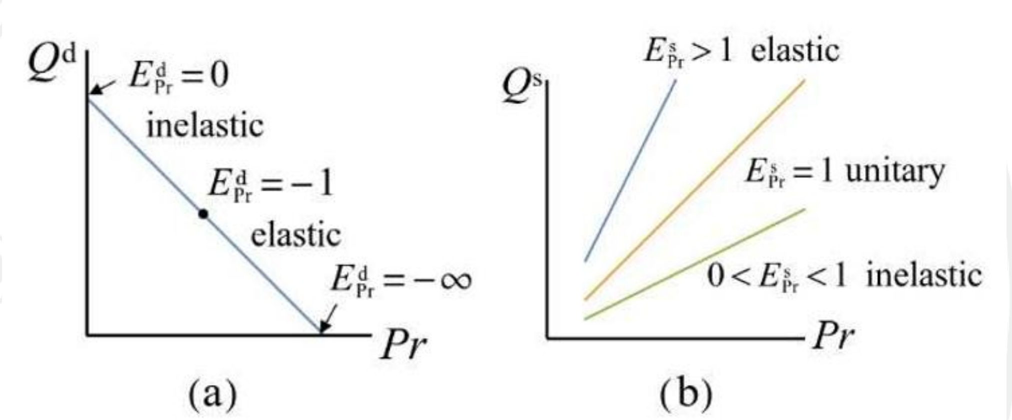

Figure 4.1 : (a) Linear Price Demand Curve (b) Linear Price Supply Curve

as in **Figure 4.1** (b). In the same spirit with the way ideal gas law is formulated, there should be a relation

$$Q^d = k_{Pr}(Pr) \cdot Q^s. \tag{4.3}$$

Working with these relations, one finds

$$(Q^d)^2 = \varepsilon_d^2 \cdot Pr^2 + k_d k_{Pr} \cdot Pr \cdot Q^{d,0}, \tag{4.4}$$

where $\varepsilon_d^2 = k_d k_s k_{Pr}$ and that

$$(Q^s)^2 = \varepsilon_s^2 \cdot Pr^2 + \left(\frac{k_d}{k_{Pr}}\right) \cdot Pr \cdot Q^{d,0}, \tag{4.5}$$

where $\varepsilon_s^2 = (k_d k_s / k_{Pr})$. Point-price elasticity of demand and supply are

$$E^d = \frac{\Delta Q^d}{\Delta Pr}\frac{Pr_0}{Q_0^d}, \qquad E^s = \frac{\Delta Q^s}{\Delta Pr}\frac{Pr_0}{Q_0^s}. \tag{4.6}$$

Since $k_s < 0$, hence ε_d and ε_s are imaginary. At market clearing price (denoted by $*$),

$$Q^{d,*} = Q^{s,*}, \tag{4.7}$$

making $k_{Pr} = 1$. The slope

$$\varepsilon_d = \frac{\Delta Q^d}{\Delta Pr} = E^d \frac{Q_0^d}{Pr_0} = k_s < 0, \tag{4.8}$$

in this case. The $\varepsilon_d, \varepsilon_s$ hence take both imaginary and real values. The inconsistency comes from existence of vertical intercept $Q^{d,0}$ in equation (4.1) but no vertical intercept in $Q^s = k_d(Q^d) \cdot Pr$, and from that the slope k_d and k_s are forced to be equal by market clearing condition, $Q^{d,*} = Q^{s,*}$.

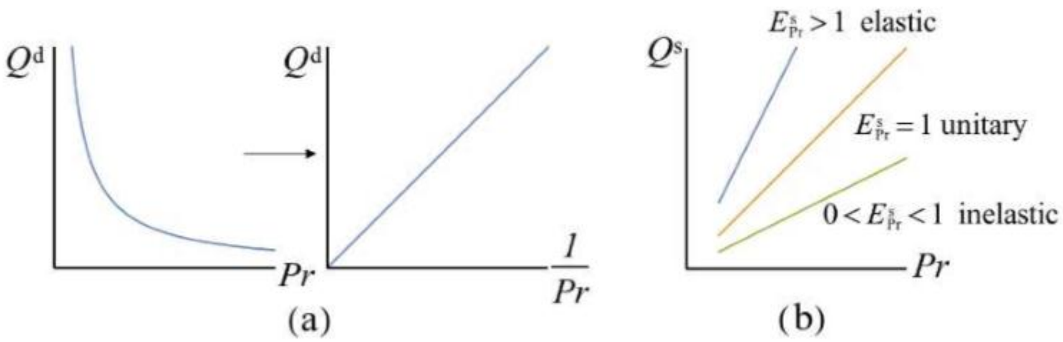

Figure 4.2 : (a) Unitary Price Demand: Hyperbolic Curve Q^d versus Pr (b) Linear Price Supply Curve

4.3 Market with Unitary Price Demand and Q^s Linear with Price

In distinct from section 4.2 , the only change is the demand function,

$$Q^d = k_s(Q^s) \cdot \frac{1}{Pr}, \tag{4.9}$$

as in **Figure 4.2**. Hence $k_s > 0$ here. Note that,

$$\delta Q^d = k_s \cdot \delta\left(\frac{1}{Pr}\right), \tag{4.10}$$

hence slope of the hyperbolic curve is

$$\varepsilon_d \equiv \frac{\Delta Q^d}{\Delta Pr} = -\frac{k_s}{Pr^2}. \tag{4.11}$$

Using this slope and equation (4.9) in equation (4.6) resulting $E^d = -1$ (i.e. unitary) at any points. In **Figure 4.2**, the hyperbolic slope is $\varepsilon_d = \varepsilon_d(Pr)$. The curve Q^d versus $1/Pr$ is linear. Following similar procedure of section 4.2 , we find,

$$Q^d = \sqrt{\frac{k_s k_{Pr}}{k_d}} \cdot \frac{Q^s}{Pr}. \tag{4.12}$$

Notice that the slope of supply curve is

$$k_d = \varepsilon_s = \frac{\Delta Q^s}{\Delta Pr} = E^s \frac{Q^s}{Pr}. \qquad (4.13)$$

In Thermodynamics, the influence (mechanical force) term is an intensive variable. We shall modify the definition of Q^d to its intensive quantity that is *price demand per household*,

$$q^d = \frac{Q^d}{N}, \qquad (4.14)$$

where N is number of household buyers. Redefining

$$q^d = k_s \cdot \frac{1}{Pr}, \qquad (4.15)$$

and

$$q^d = k_{Pr} \cdot Q^s, \qquad (4.16)$$

we see that k_{Pr} is a constant N^{-1}. We hence succeed in writing $f(X, Y, T) = 0$ as

$$f(Q^s, q^d, Pr) = 0. \qquad (4.17)$$

This makes

$$q^d = K \frac{Q^s}{Pr} \quad \text{or} \quad Q^s = \frac{1}{K} q^d \, Pr, \qquad (4.18)$$

where $K \equiv \sqrt{k_s/(\varepsilon_s N)}$ playing similar role to the gas constant R or k_B combining with the specific property, e.g. compressibility factor $Z(T)$ in real virial gas which encodes all specific and realistic properties of system. For ideal system $Z = 1$. Hence for real systems the factor is to be found empirically employing sophisticated statistical methods. **Figure 4.3** presents analogous surface of ideal gas EoS and $f(Q^s, q^d, Pr) = 0$, as

$$X \equiv Q^s, \quad Y \equiv q^d \quad \text{and} \quad T = Pr. \qquad (4.19)$$

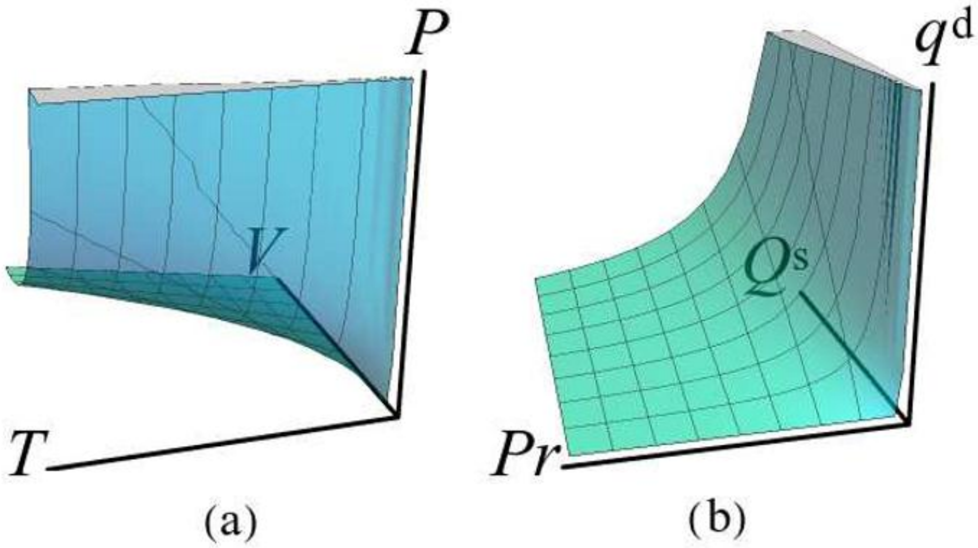

Figure 4.3: (a) EoS of Ideal Gas as a Two-Dimensional Surface (b) Surface (equation (4.18)), a Possible EoS of Market with Unitary Price Elastics of Demand

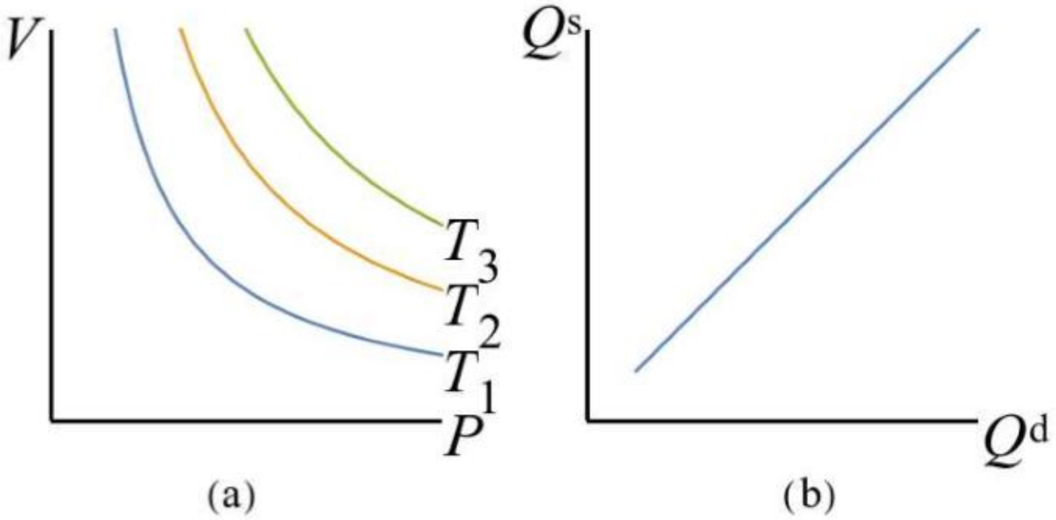

Figure 4.4: (a) Isothermal Curve (b) Isoprice Curve

4.4 Chapter Conclusion and Critics

We try to find thermodynamical coordinates of a market. We propose the zeroth law such that the price plays the role of temperature. The EoS coordinates (Q^s, Q^d, Pr) are hypothesized at first. Market with linear price elasticity of demand is inconsistent due to market clearing (equilibrium) condition and form of equation. Inconsistency of ε comes from the need that $Q^d = Q^s$ whiles having two different values of slope, k_d, k_s. In case of market with unitary price elasticity of demand, it is possible to derive a constraint surface with an additional idea that the demand is considered to be intensive.

As we consider equilibrium state, quantity of goods in transaction should be denoted with only one single variable, Q, not to be considered separately as two forces of willing to sell and of willing to buy. As a result, one cannot draw a set of *isoprice curves* in the same spirit as isothermal curves as seen in **Figure 4.4**. In **Figure 4.4** (a) T is an exogenous variable of the $V\ P$ plane. In **Figure 4.4** (b) the market is with unitary price demand. Pr is an exogenous however different prices do not give different curves but instead giving different value of $Q^s = Q^d$ via $q^d = (1/N)Q^s$. This is distinct from that of hydrostat system where P and V are hyperbolic to each other. One of these variables might not be the correct coordinates. There are internal (the system) and external (the environment) parts to exchange heat and work hence resulting change in internal energy. The situation of market case is different. Transactions to form clearing price in a market are completed by buyers and sellers who are both internal agents. Forming an equilibrium price does not tell us the total wealth of the market while forming an equilibrium temperature does tell us how much internal energy the system possesses.

We also notice that in (P, V, T) system, P results in change of V. Both are different quantities. In simple paramagnetics, magnetic field, B_0 induces changes in magnetization $\mathcal{M}$, via the EoS, $\mathcal{M} = \mathcal{C}(B_0/T)$. The physical quantities B_0 and $\mathcal{M}$ are similar in nature. Effect of B_0 is *amplified* by factor $\mathcal{C}$ (Curie constant) to induce $\mathcal{M}$. In unitary market, intensive influence q^d is similar to Q^s by nature of their physical quantities. From equation (4.18), $(1/K)$ looks like the amplification factor for q^d to induce Q^s. Here we have explored possibility towards construction of an

EoS for a market and we gain some insights and learn some shortcomings of the paradigm developed here to proceed in future construction.

Validity of the proposed EoS (equation (4.18)) in this chapter will be disproved in the next chapter and some additional critics.

CHAPTER 5

EFFECT STRUCTURE DIAGRAM

Physical laws have some deeper physical insights that, at present, can not be represented by the existing mathematical equations. The variables' role of being the cause to other variables and role of being affected by the others are not clearly stated. In an EoM, for instance, Newton's second law, force is the cause and the effect is on the velocity. Similarly in Ohm's law, potential difference is the cause and the effect is on charge quantity moving through an area. The physical property in the Newton's law is inertial mass while in Ohm's law the physical property is resistivity. Similarities of the two laws give no surprise that mechanical vibration shares the same mathematics as of the RLC circuits.

In an EoS, unlike Newton's or Ohm's laws, there are three variables, temperature (T) with extensive coordinate (X) and intensive coordinate (Y). As we do partial derivative, the cause and effect must be considered within a pair of variables keeping the rests constant. Cause and effect in this way is known as endogenous effect which is effects of internal variables existing explicitly in a *setup model*. A change in other variables that are not in the independent variables of the model are typically known as exogenous effects. If concept of the model is not suffciently clear, it is not easy also to have clear identification of which variables are endogenous or exogenous. Therefore we give definition of *truly endogenous effect* as a set of independent variables mapping with non-composite function to a set of independent variables. We analyze on how well-known mechanical pairs of EoS are expressed. We propose a diagram describing pattern of truly endogenous effects among the EoS variables and propose some criteria to check naturalness of an EoS. Truly endogenous effects in some EoS are discussed here so that one can classify types of the known EoS into two classes and the effect structure is expressed in form of a diagram, namely the *effect structure diagram* of an EoS. It addresses how *an EoS* should look like if we are to construct a new EoS empirically from a system of consideration.

A formal way in describing the classes of EoS is proposed so that we can comment and justify the EoS status of an empirical relation. Knowing these structures of the EoS could hint the way to construct other EoS in analogous. In this chapter, we check the naturalness of economics EoS proposed in CHAPTER 4 or in Gumjudpai (Gumjudpai, 2018) and disprove it. Having consideration of the EoS of an economics system could lay out some novel approach to economics employing thermodynamics. The results of this chapter bases on Gumjudpai and Setthapramote (Gumjudpai & Setthapramote, 2019).

5.1 Roles of Thermodynamics Coordinate Variables in an EoS

5.1.1 Endogenous and Exogenous Effects

Endogenous and exogenous variables are common economics terms. Consider set of real value variables, $\{x_1, \dots, x_n\}$ and let there be a function,

$$x_1 = f(x_2, \dots, x_i, \dots, x_n). \tag{5.1}$$

$(x_2, \dots, x_i)$ are endogenous variables of f for constant values of all the rest variables, i.e. $(x_{i+1} = a_{i+1}), \dots, (x_n = a_n)$. That is

$$x_1 = f(x_2, \dots, x_i, a_{i+1}, \dots, a_n) = f_2(x_2, \dots, x_i). \tag{5.2}$$

Hence $(x_2, \dots, x_i)$ have endogenous effect on x_1. If $(a_{i+1}, \dots, a_n)$ change their values, these variables $(x_{i+1}, \dots, x_n)$ are said to be exogenous variables and have exogenous effect on x_1 This typically happens when ones try to deal with data obtained from some phenomena with a basis of thought that the factors of change that affect x_1 in the model are only $x_2, \dots, x_i$ which are considered as the only set of independent variables in framework of the model. When doing regression analysis, all other $(a_{i+1}, \dots, a_n)$ are regarded as constant of the regression. If we change conceptual framework of the model to include some exogenous variables to the model, for example, including of $x_{i+1}$to the model, hence x_{i+1} is endogenous and we obtain

$$x_1 = f(x_2, \dots, x_i, x_{i+1}, a_{i+1}, a_{i+2}, \dots, a_n) = f_2(x_2, \dots, x_{i+1}). \tag{5.3}$$

In economics, exogenous change is sometimes known as shock of demand or supply functions in markets of different types of product, e.g. labours, loanable funds, etc.

5.1.2 Role of Variables

Thermodynamics coordinates are not equivalent although each of them is axis of 3dimensional thermodynamics space. The three variables are T, Y (intensive coordinate) and X (extensive coordinate). Temperature is a fundamental thermodynamics coordinate since it is laid by the 0th law of thermodynamics. Pair variables $(X, Y\ )$ is dubbed the mechanical pair because they are from the class of physics derived from least action principle. Mechanical pair must form a work term, $\delta W\ =\ Y\,\mathrm{d}X$. Examples of mechanical pair are such as followed. In hydrostatics system, X is the volume $V\ , Y$ is $-P$ (minus sign of pressure). In paramagnetics substance, X is the magnetization $\mathcal{M}, Y$ is an external magnetic field intensity B_0. In one-dimension elastic string, X is $\ell \equiv\ x^2/2$ and Y is $f\ \equiv\ F/x$ (force per length). In a two-surface rectangular thin film with length x and with constant width l, X is area $A\ =\ lx$ and Y is 2Γ where $\Gamma = F/l$ is the surface tension. In a reversible electrolytic cell, X is charge q and Y is electromotive force ε which is an electrical energy per unit of charge. In dielectric substance, X is total electric dipole moment $\mathcal{P}$ and Y is an external electric field intensity E.

1) Extensive coordinate, X

Considering only endogenous effect, in some cases, X has a passive role, i.e. it takes an effect of other influence that causes changing in X. For instance, the volume V can not change without the effect of pressure P. In paramagnetics substance, magnetization $\mathcal{M}$ can not change unless being an effect of magnetic field intensity, B_0 or T. In this case X (that is $\mathcal{M}$) can take active role, i.e. changing in $\mathcal{M}$ (as an effect of B_0) can cause changing in T.

2) Intensive coordinate, Y

Intensive variables Y are physical quantities with nature of influence e.g. force, power or energy but per unit of the quantity or relating quantity which it has an endogenous effect on. Example of Y , that is defined with influence per unit of quantity X it has effect on, is the electromotive force $\mathcal{E}$ in reversible (ideal) electrolytic cell. $\mathcal{E}$ is energy per unit charge (X is charge q) and $\mathcal{E}$ causes the effect on charge quantities q in motion. The other cases are slightly different that Y is an influence per unit of a quantity related to X (instead of per unit of X). For examples, in hydrostatics system, Y is $-P$ which is force per area, F/A. The extensive X is volume V . Volume V is related to area A. In elastic string, Y is $f = F/x$ but X is $\ell \equiv x^2/2$. In two-surface thin film, $Y \equiv 2\Gamma = 2F/l$ and $X \equiv A = lx$. In case of paramagnetics, Y is B_0 which has a spirit of force per magnetic charge (not exist in reality) which is related to the extensive $\mathcal{M}$. In dielectrics, Y is electric field intensity E which is a force per unit charge. Charge is related to Y variable that is the total electric dipole moment $\mathcal{P}$ by $\vec{\mathcal{P}} = \sum q\vec{d}_i$ where $\vec{d}_i$ points from negative to positive charge. Although Y has active role as influence, it also has passive role, i.e. T can cause endogenous changes in $-P$ in a hydrostatic system.

3) Temperature, T

T tells us thermal state of a system. T must have both active and passive roles. In hydrostatic system, T can endogenously affect Y (that is $-P$) and reversely P can endogenously affect T. In paramagnetics system, T can endogenously affect X (that is $\mathcal{M}$) and reversely $\mathcal{M}$ can endogenously affect T.

5.2 Truly Endogenous Effect

In this work, we try to find a function that depends on *truly endogenous variables* which we define as a set of independent variable mapping with non-composite function to a set of independent variable. For example, we know that V depends endogenously only on P. Temperature T can not affect V directly but via the pressure P, i.e. temperature affects the pressure and the pressure affects the volume afterwards. Hence to write a function of an EoS as

$$V = V(P,T), \tag{5.4}$$

does not make good sense because indeed it is

$$V = V \circ P(T) = V\big(P(T)\big). \tag{5.5}$$

Instead, for truly endogenous effect, it should be expressed as special notation of function for *truly endogenous function* as

$$V = \tilde{V}(P), \tag{5.6}$$

and the others are

$$P = \tilde{P}(T) \qquad \text{and} \qquad T = \tilde{T}(P). \tag{5.7}$$

or inverse function of these functions. The truly endogenous function does not describe cause and effect of variables. We hence propose a simple diagrammatic viewpoint next.

5.3 Proposition of Effect Structure Diagram for EoS

5.3.1 Hydrostatics and Paramagnetics as Representations of Other EoS

Hydrostatics (e.g. gases and solids) and paramagnetics systems are two simple systems of which the EoS are well-known. Moreover the known EoS of many other systems are similar to either the hydrostatics or the paramagnetics. Systems of elastic string and two-surface thin film are similar to the hydrostatics system due to their mechanical work characters. The reversible electrolytic cell and dielectric substance are similar to the paramagnetics due to their electromagnetic work characters. Hence, the two systems are good representatives of the two types of EoS.

5.3.2 Class I and Class II Effect Structure Diagrams

We analyze truly endogenous effect of coordinate variables in the two EoS using arrows connecting from variable that is the cause to the variable that is affected by the cause. That is to say one variable has truly endogenous effect to the other that has passive role. We classify them into two classes as in **Figure 5.1**. Expressing hydrostatics EoS and paramagnetics EoS (Curie's law) in X, Y variables and let k be a constant magnification factor of combined effects of Y and T,

Case I: $$V = \frac{nRT}{P} \qquad \Rightarrow \qquad X = k\left(\frac{T}{Y}\right), \qquad (5.8)$$

Case II: $$\mathcal{M} = \mathcal{C}\frac{B_0}{T} \qquad \Rightarrow \qquad X = k\left(\frac{Y}{T}\right), \qquad (5.9)$$

where k is an arbitrary magnification factor to the combined result of Y and T in $f(Y,T)$, e.g. for Class I, $f(Y,T) = T/Y$. In Class I, T is proportional to X, i.e. having enhancing effect to X but Y is inversely proportional to X, i.e. having reducing effect to X. Situations are reversed in the case of Class II. In **Figure 5.1**, the diagrams represent Class I and Class II of EoS. Hydrostatics system is an example of effect structure in Class I type EoS. Temperature T can affect pressure P and so reversely. P

can affect volume V . However V can not affect any other variables. On the right, for generalization, X and Y are written in the diagram. Paramagnetic system is an example of effect structure in Class II type EoS. Temperature T can affect magnetization $\mathcal{M}$ and so reversely. External magnetic field intensity B_0 can affect $\mathcal{M}$. However B_0 can not be affected by other variables.

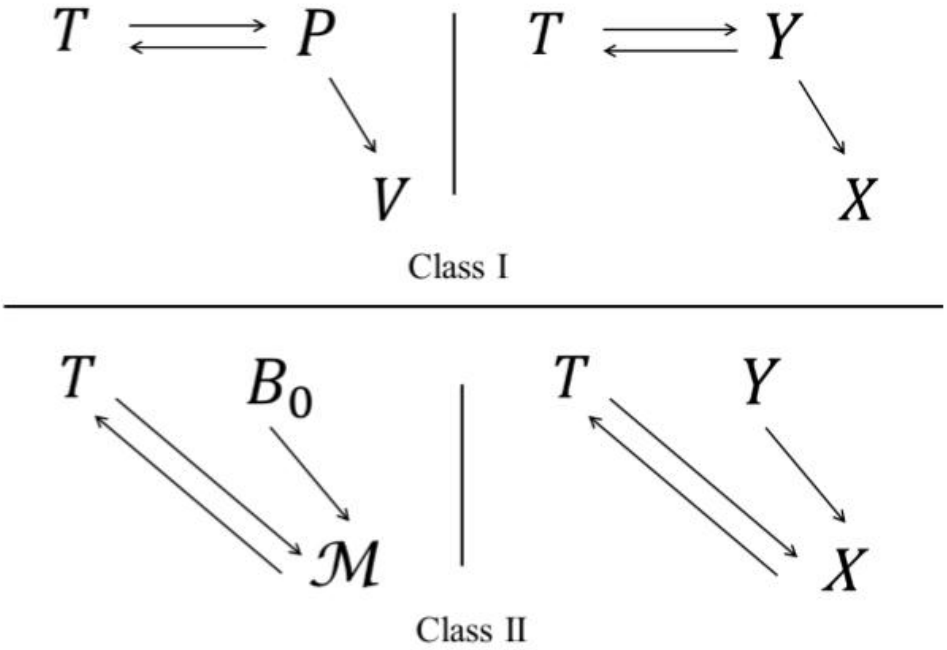

Figure 5.1: Diagram Showing Truly Endogenous Effect in EoS Variables of Hydrostatics System (Upper) and Paramagnetics System (Lower).

5.4 Testing of a Proposed EoS in Section 4.3

With aspects we gain from analysis of roles and relation between coordinate variables in an EoS, we will take these aspects as basic *criteria for testing of whether an empirical relation could have a status of an EoS*. We try with EoS of a market with unitary price demand and supply as linear function of price as previously proposed in section 4.3 (Gumjudpai, 2018). The proposed EoS, $g(Q^s, q^d, Pr) = 0$ is the equation (4.18)

$$Q^s = \frac{1}{K} q^d Pr = 0,$$

where Q^s is supply quantity, q^d is demand quantity (Q^d) per number of consumer (N) and Pr is the price of commodity. All are measured at equilibrium and the market is effcient and information symmetric. The extensive X is Q^s, the intensive Y is q^d. T is Pr by the assumption that price equilibrium is analogous to thermal equilibrium, i.e. temperature identifies thermal equilibrium as of the 0th law of thermodynamics and price identifies the market's price equilibrium as of the analogous 0th law. We draw truly endogenous effect diagram employing empirical relation of price mechanism as in **Figure 5.2**. Written in term of X, Y variables, both Y and T are proportional to X.

$$Q^s = \frac{1}{K} q^d Pr \qquad \Longrightarrow \qquad X = k\, YT. \qquad (5.10)$$

We shall see whether this equation should be classified as the Class III EoS. We criticize in two points as follow.

1) In Class I and II diagrams, there are three arrows of the truly endogenous effect but the diagram in **Figure 5.2**, which is the proposed EoS, there are four arrows. There are two incoming arrows to Pr (analogous to T) and two outgoing arrows from Pr. We try a specific case of the classical school of economics, that is the *Say's law* (Sexton, 2011). The law states that supply quantity Q^s dictates demand quantity Q^d. According to the law, there is no supply surplus left in the market. If Q^s only affects Q^d and it does not affect Pr but instead Pr can affect Q^s, we hence have diagram as in **Figure 5.3** with four arrows. The consequence is that as Q^s increases exogenously, q^d increases. As q^d increases, the price increases truly endogenously. Higher price reduces q^d truly endogenously but increases the supply Q^s truly

endogenously. As we have higher Q^s, then q^d should be higher. However it does not. This is because the price is now higher and q^d is less. The law seems to be wrong but it is not the case. We must remember that the effect of Q^s to q^d according to Say's law must be exogenous, not truly endogenous. How supply can increase demand is because more production results in more income of labour. Labour people are consumers hence richer consumers have more demand q^d. This is exogenous effect of the income to q^d. Arrows in the context we propose must be truly endogenous therefore there should not be an arrow from Q^s to q^d in the diagram. If we want to fix the diagram by removing the arrow from Q^s to q^d, we are left with three arrows as of the Class I and II's diagrams. Producers then can not influence the price and become price takers. Producing less does not increase the price and this is not true.

2) At price equilibrium, $Q^s = Q^d$. As a result,

$$q^d - \frac{Q^s}{N} = 0, \tag{5.11}$$

which is a constraint $h(q^d, Q^s) = 0$. Combining with the proposed EoS (4.18), there is only one degree of freedom left for the system. Quasi-static evolution is a process along one-dimensional geometrical object. Hence it is clear that the equation (4.18) constrained with equation (5.11) is not two-dimensional equilibrium surface but a line.

$$\begin{tikzcd}
Pr \arrow[r, shift left] \arrow[dr, shift left] & q^d \arrow[l, shift left] \\
 & Q^s \arrow[ul, shift left]
\end{tikzcd}
\qquad\Bigg|\qquad
\begin{tikzcd}
T \arrow[r, shift left] \arrow[dr, shift left] & Y \arrow[l, shift left] \\
 & X \arrow[ul, shift left]
\end{tikzcd}$$

Figure 5.2: Truly Endogenous Effect Diagram of the Proposed EoS of a Market with Unitary Price Demand and Supply as Linear Function of Price. On the Right is the Diagram Represented with the Extensive and Intensive Coordinates X and Y .

$$\begin{tikzcd}
Pr \arrow[r, shift left] \arrow[drr] & q^d \arrow[l, shift left] & \\
& & Q^s \arrow[ul]
\end{tikzcd}$$

Figure 5.3: Trying to Apply Say's Law to the Diagram by Allowing an Arrow from Q^s to q^d Makes a Wrong Diagram. This is Because, in Say's Law, the Supply Affects Demand Exogenously, not Endogenously.

5.5 Chapter Conclusion and Critics

We consider major features of thermodynamic coordinate variables of various physical systems. We analyze and criticize on roles of these variables as they are intensive and extensive coordinates and temperature which are variables in an EoS. In doing this, we aim to find a central concept in constructing an EoS from other systems such as an economics system and to find some criteria for judging if an empirical equation could take a status of an EoS. We propose diagram presenting truly endogenous effect among these variables so that they can be classified. This sets a foundation in constructing and testing of an EoS, of particular interest here, an economics system. We consider thermodynamics approach of economics in distinct ways from other previous works (Lisman, 1949; Saslow, 1999; Smith & Foley, 2008) which have initial setting from optimal concepts of variables. Here we take Carathéodory's axiom which concerns much on the existence of the EoS derived from empirical facts as initial setting. We found that the EoS can be classified into two classes. We use the concepts we found here in analyzing the proposed EoS of a market with unitary price demand and linear-in-price supply functions proposed in section 4.3. We found that the proposed EoS is not a new class and is not even an EoS due to the price equilibrium condition which is a constraint condition. The constraint reduces one degree of freedom of the system resulting only one degree of freedom left therefore the proposed equation is not an EoS surface but only a line.

CHAPTER 6

DEMAND-SIDE EQUATION OF STATE

In this chapter, we first set the rules of EoS which are fundamental for further analysis. Next we derive the demand-side EoS which is crucial in studying demand system using formal thermodynamical approach. Next we further formulate the EoS for a system of consumers in a perfect competitive market. Finally, based on the EoS, we propose how to apply Monte-Carlo simulation in analyzing the demand system. Our view now is to consider the demand side independently from the supply side. We do not consider market as a whole as mistakenly done in CHAPTER 4.

6.1 Rules of EoS Effect Structure

Aspects noticed by our study and shall be applied as criteria of the status of an EoS, $g(X, Y, T) = 0$ of a system. These come as some novel rules for an empirical relation to be considered as an EoS. These rules are

1) Number of the arrows are three.
2) There is at least one arrow pointing $Y \rightarrow X$.
3) Only T and(or) Y (apart from causing truly endogenous effects) can also be considered as exogenous influence or a shock.

One might argue that X (e.g. volume) can be independent variable in the equation that related Y and T, i.e. $Y = Y\,(T(X)) = Y \circ T(X)\ or\ T = T(Y\,(X)) = T \circ Y\,(X)$ so that changing X might look like exogenous effect. However, for a system in reality, given a value of X at beginning, one can not change value of X unless with truly endogenous effect from Y and T.

6.2 Demand-Side Equation of State

In strictly-systematic deriving of thermodynamics following Carathéodory axioms (Buchdahl, 2009; Münster, 1970), clear differentiation between empirical facts (e.g. pressure, volume) and theoretical entities (e.g. temperature, heat) must be considered. Empirical temperature is defined with mechanical pair as in equation (3.37),

$$t = t(X, Y),$$

such that the EoS is constructed. Internal energy, U and the first law is defined purely empirically. Hence existence of the EoS is the beginning step in strictly deriving of thermodynamics. We adopt the Carathéodory approach and use it in building foundation of thermodynamics formulation of economics.

6.2.1 Price and Temperature Are Not Analogous

In this chapter, we notice that, although price Pr and T have similar theoretical origin that they are abstract and they identify thermal and price equilibriums, however Pr can not be regarded as temperature. Thermal equilibrium is much different from price equilibrium. In thermal equilibrium, the system has only one spatial sector. Energy are exchanged between molecules therefore U is conserved. In price equilibrium, there are two sectors, i.e. consumers and producers (demand side and supply side). Commodities are exchanged with other assets (e.g. money) and neither of commodities nor money are conserved in *each* sector but exchangeable. Here unlike CHAPTER 4 we do not consider market as a system, but instead we consider consumer sector as a system and we will construct a demand-side EoS. It can be picturized of internal part of a piston as demand side (buyer) and external part of the piston as supply side (seller) interacting via balancing of internal and external pressures.

6.2.2 Demand-Side Economics Version of Thermal States

The system of interest is now a group of consumers in small economy, i.e. the demand side of which the process is to buy only. The commodity is a *fixed asset* which is *reusable* and *non-disposable*. Transfers of valuable assets (e.g. money) is considered as energy transfers analogous to the work term (δW) in thermodynamics. The mechanical pair is therefore,

$$Pr \qquad \text{(as intensive coordinate } Y\text{)},$$

and demand quantities

$$Q^d \qquad \text{(as extensive coordinate } X\text{)}.$$

If there is an economic quasi-static expansion process which is a quasi-static change in Pr and Q^d in perfect competitive market, hence following Carathéodory axioms, we make empirical axiom of existence of infinite and arbitrary economic thermal states,

$$t\left(Pr_1, Q_1^d\right) = \varphi_1 \qquad \text{and} \qquad t\left(Pr_2, Q_2^d\right) = \varphi_2 \qquad \text{and so forth.} \tag{6.1}$$

Each state shall be identified with personal total value (φ) which is analogous to temperature T. The demand-side economy's total wealth $\mathcal{W}$ is the sum of demand-side economy's total possession of liquidity asset (e.g. money) and demand-side economy's total possession of fixed assets plus demand-side economy's total utility earned from using and owning of fixed assets (this is our new definition and is not the same as in section 3.3 which is the work of Saslow (Saslow, 1999)). All these quantities are the sum of the quantity that each consumer possesses. The *personal wealth* is hence the total wealth per buyer,

$$\varphi = \frac{\mathcal{W}}{N}, \tag{6.2}$$

where N is the number of consumer or household. Personal total value takes a role of *demand-side microeconomic temperature*. This is similar to monatomic ideal gas that $T = 2U/(3Nk_B) = 2U/(3nR)$ whereas N is number of particle and n is molar number. Economics does not have as many fundamental units as in physics. In economics, there are units of value (e.g. in money unit), number of goods and number of buyers only. Hence there might not be such constants k_B or R here to balance the equation.

6.2.3 Demand-Side Economics Version of Thermal Equilibrium

In thermodynamics, changing of adiabatic wall to diathermic wall allows heat exchanges and takes the two phases to thermal equilibrium as T takes the same values. Heat at higher temperature is ready to go to lower temperature so that thermal equilibrium is established. So as economic heat goes from higher φ to lower φ or to interpreting as two groups of consumers (each with φ_1 and φ_2) are allowed averagely co-utilize the fixed asset or share the usages of each group's all accumulated fixed assets. This situation is like when two groups of people become one bigger group. Although, the liquidity asset (e.g. money) is not yet to share but in long run, as one single family or closed household, money is likely to distribute to everyone either fast or slowly. The long-run money distribution is not transferred by trading but by giving.

6.2.4 Demand-Side Economics Version of Mechanical Work

Since price, Pr is Y here and in perfect competitive market price is a marginal revenue, MR which is equal to marginal cost of production, MC. Price takes a role of influence (on buying or selling) per unit of goods. The demand-side economics work is

$$\delta W_{\mathrm{Dm}} = -Pr\mathrm{d}Q^d. \qquad (6.3)$$

The demand-side economy's total wealth, $\mathcal{W}$ plays the same role as internal energy U, hence adiabatic first law of thermodynamics,

$$\mathrm{d}U = \delta W = Y\mathrm{d}X, \qquad (6.4)$$

should analogously read

$$\mathrm{d}\mathcal{W} = \delta W_{Dm} = -Pr\mathrm{d}Q^d. \qquad (6.5)$$

Transaction of liquidity asset (e.g. money) of a group of consumers reduces demand-side economy's total wealth (total wealth of the whole consumers) by $\mathrm{d}\mathcal{W}$ and the value of transaction is negative (i.e. expense) at value of $Pr\mathrm{d}Q^d$.

6.2.5 As $\boldsymbol{Q^d \neq 0}$, Market Does Not Stop But Static.

One needs to be notified that, in economy, when there is non-zero demand quantity, there are movements in market's activity. The buying and selling happen all the times. A market system at static economic equilibrium contains microscopic transactions or exchange of money and goods all the times. This is just as at thermodynamics equilibrium, there are interactions between microscopic particles all the times in a closed piston filled with gas or in a paramagnet. Only when $X = 0$, e.g. volume or magnetization is zero, there is no particle's thermal interaction.

In thermodynamics when extensive variable is unchanged (e.g. constant volume or constant magnetization) there are no energy transfer as work term but one must keep in mind that particles are still in interaction, exchanging their kinetic energy. Hence in economy, when Q^d is constant that is $\mathrm{d}Q^d = 0$, it is easy to misunderstand that all customers stops buying or selling. Indeed, we must consider that buying and selling happen all the times except when $Q^d = 0$.

6.2.6 Effect Structure and Class III EoS

Following concepts and criteria in section 6.1, we propose an EoS,

$$g(Q^d, Pr, \varphi) = 0, \qquad (6.6)$$

written as

$$Q^d = kf(Pr, \varphi), \qquad (6.7)$$

or for generalization,

$$X = kf(Y, T). \qquad (6.8)$$

In perfect competitive market, we know from facts that Q^d is typically inversely proportional to Pr but it is proportional to φ. Diagram in **Figure 6.1** is based on empirical fact and it obeys all rules given in section 6.1. Notice that exogenous effect can come from Pr and φ as required. The diagram is distinct from the Class I and II hence it is categorized as Class III. Personal wealth φ can affect demand quantity Q^d. Price Pr can affect Q^d and so reversely. φ can only affect Q^d.

6.2.7 Proposing Two Types of Economic Equilibrium

Referring to section 2.2.7, in thermodynamics of a closed system, there are two types of equilibrium:

1) mechanical equilibrium (equilibrium of pressure, P)
2) thermal equilibrium (equilibrium of temperature, T)

which tell us that characteristics of equilibrium are due to two causes of change that are temperature and pressure. In a closed demand-side economy system, as suggested by effect structure diagram there are two types of equilibrium,

1) price equilibrium (equilibrium of price, Pr - analogous to mechanical equilibrium)

2) personal wealth equilibrium (equilibrium of personal wealth, φ – analogous to thermal equilibrium)

These hint us further construction of the first law of thermodynamical economics. In summary using concept of thermodynamics, we can define relationship between variables of demand system in the light of effect structure diagram. This leads to application of thermodynamics in analyzing of the demand system. In the next two sections, we apply specific methods in physics to the EoS. To provide numerical examination of the EoS and to provide guideline of further economics application, the method considered is the Monte-Carlo simulation.

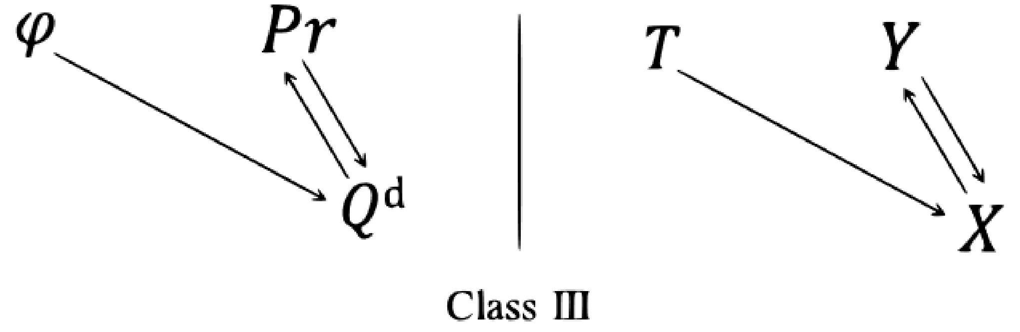

Figure 6.1: Effect Structure Diagram of a Group of Consumers in Perfect-Competitive Market (Class III Type EoS).

6.3 Using EoS in Modeling of Demand Function for Econometrics

EoS is empirical law, known in social science as a model. It can be constructed with observation or by doing a thought experiment which gives a model of a perfect or ideal system. That is to say one can have empirical law of a perfect or ideal system that does not exist in the real world.

6.3.1 Virial EoS of Real Gas

Towards a realistic prediction of an EoS, statistical method is employed. As an example, we will take the *virial EoS of real gas* as a case of study. Recall ideal gas EoS (where specific molar volume $v \equiv V/n$),

$$Pv = RT$$

which can be rewritten as

$$\frac{PV}{nRT} = Z(v), \tag{6.9}$$

where $Z = 1$ for the ideal gas. When $Z \neq 1$, the factor encode *realistic properties* of the gas. Typically $Z = Z(T)$ is an infinite series

$$\frac{PV}{nRT} = Z(v) = 1 + \frac{B}{v} + \frac{C}{v^2} + \frac{D}{v^3} + \frac{E}{v^4} + \cdots. \tag{6.10}$$

Using data from experiments, one can determine the coefficients $B, C, D, E, \ldots$. The more precise parameters we have, prediction using the EoS has less and less error.

6.3.2 Modeling of Demand-side EoS with the Basics of EoS

In econometric modeling, one need to specify assumption of the modeling and propose a mathematical model. For example, dependency of Y due to X_1, X_2, X_3, X_4 might be proposed as

$$Y = \beta_1 X_1 + \beta_2 X_2^2 + \beta_3 X_3 + \beta_4 \frac{1}{X_4} + \alpha\,, \tag{6.11}$$

where β_n and α are parameters to be found from regression analysis of data (Gujarati, 2009). Modeling Y as function,

$$Y = f(X_1, X_2, X_3, X_4), \tag{6.12}$$

does not give any insights of which variables are truly endogenous or exogenous. What we attain in this thesis proposes the concept of EoS in modeling of demand function. Although it is consider at fixed time equilibrium without the effect of time lagging, it gives a good reason of which variables are the truly endogenous, leaving other exogenous effect to be analyze in the other sets of equations. Considering the EoS equation (6.7),

$$Q^d = kf(Pr, \varphi).$$

Let us suppose unitary price demand function as truly endogenous function (see section 5.2),

$$Q^d = \tilde{Q}^d(Pr) \tag{6.13}$$

We suppose demand function as truly endogenous function of φ as

$$Q^d = \tilde{Q}^d(\varphi) \tag{6.14}$$

where we use α, β_n, γ as parameters of the regression. In this small economy, to be able to buy a unit of fixed asset, a consumer's value of average personal liquidity asset (money own asset or borrowing) M is the purchasing power. As we consider the whole economy in average, hence it is possible that $0 < Q^d < 1$ in average. There are also variations of modelling of M, for examples, we might have

1) $\mathrm{M}(\varphi) = \alpha\varphi,\ 0 < \alpha < 1$

Consumer's cash or liquidity is a constant fraction of personal wealth (φ). We assume constancy of α here such that personal liquidity depends on personal wealth φ only. That is the economy maintains the fraction of liquidity with respect to the economy's total wealth.

2) $\mathrm{M}(\varphi) = \varphi - \rho$

Consumer's cash or liquidity is a difference of personal procession of wealth (φ) and personal possession of fixed asset plus utility earned from using and owning of fixed assets (ρ) which may not be constant.

We have to take into account that buying of fixed asset should be less and less as the quantity of fixed assets in the small economy are greater and greater. This has to resemble the concept of *diminishing marginal utility* of neoclassical economics. Let us take a look at some possible models:

1) First example:

$$Q^d = \frac{\beta_1}{Pr} + \beta_2 \mathrm{M}(\varphi) + \alpha. \tag{6.15}$$

In such model, it is not convenient that there is no term reflecting comparison between the value of Pr and $\mathrm{M}(\varphi)$. Moreover Q^d grows with $\mathrm{M}(\varphi)$ hence does not follow the concept of diminishing marginal utility.

2) Second example:

$$Q^d = \beta_1 \ln\left(\frac{\mathrm{M}(\varphi)}{Pr}\right) + \beta_2(\mathrm{M} - Pr) + 1, \tag{6.16}$$

where $\beta_2 < 0$. At greater M, Q^d decreases. The problem exists when $\mathrm{M} < Pr$ in second term. The sign swap contributes to more Q^d!

3) Third example:

$$Q^d = N \frac{\mathrm{M}(\varphi)}{Pr} Z(q^d), \tag{6.17}$$

where N is number of consumers in the economy. This is analogous to the virial gas in equation (6.10). For M $= \alpha\varphi$, we express it as approximated at first order,

$$\frac{Pr\, q^d}{\alpha\varphi} = Z(q^d) \simeq 1 + \frac{B}{q^d}, \qquad (6.18)$$

where as $q^d \equiv Q^d/N$ is average personal demand. Taking logarithm, hence

$$\ln(q^d) \simeq \ln\alpha + \ln\varphi - \ln Pr + \ln\left(1 + \frac{B}{q^d}\right), \qquad (6.19)$$

which is still difficult to fit in the framework of econometric regression. Realistic aspect of data (property) is to be decoded from B.

6.3.3 Modeling of Demand-side EoS with Total Differential

Now we are certain that we have EoS in general form, $q(Q^d, Pr, T) = 0$. If we do not pretend to presume the form of the EoS before hand, but instead we naively begin with a common method in thermodynamics, the total differential method, we shall express,

$$\mathrm{d}Q^d = \left(\frac{\partial Q^d}{\partial\varphi}\right)_{Pr} \mathrm{d}\varphi + \left(\frac{\partial Q^d}{\partial Pr}\right)_{\varphi} \mathrm{d}Pr. \qquad (6.20)$$

Defining coefficients,

$$\beta_P r \equiv \frac{1}{Q_0^d}\left(\frac{\partial Q^d}{\partial \phi}\right)_{Pr} = \frac{E_\phi^d}{\varphi_0}, \qquad (6.21)$$

$$\kappa_\varphi \equiv -\frac{1}{Q_0^d}\left(\frac{\partial Q^d}{\partial Pr}\right)_{\varphi} = -\frac{E_{Pr}^d}{Pr_0}, \qquad (6.22)$$

where E_ϕ^d and E_{Pr}^d are elasticities of demand to personal wealth and demand to price. In simplest case, elasticities may be assumed constant (Elasticities could depend on other factors as similar to the factors in EoS of simple solid). It is straightforward to write,

$$Q^d(Pr, \varphi) = Q_0^d\left[1 + \beta_{Pr}(\varphi - \varphi_0) - \kappa_\varphi(Pr - Pr_0)\right], \qquad (6.23)$$

which is the EoS for the demand-side system. Further we define

$$Y \equiv \frac{Q^d}{Q_0^d}, \tag{6.24}$$

$$X_1 \equiv \varphi - \varphi_0, \tag{6.25}$$

$$X_2 \equiv Pr - Pr_0. \tag{6.26}$$

Our EoS is hence

$$Y = 1 + \beta_{Pr} X_1 - \kappa_{\varphi} X_2, \tag{6.27}$$

and our econometric model for the EoS is hence,

$$Y = 1 + \beta_{Pr} X_1 - \kappa_{\varphi} X_2 + u, \tag{6.28}$$

where u is an error term. We can translate the equation which provides guideline to link EoS in thermodynamics with regression analysis in econometrics. This enables further applying econometrics methods in analyzing of the EoS in thermodynamics. However in some cases we can not find actual data, hence performing Monte-Carlo simulation is useful.

6.4 Monte-Carlo Simulation of the Demand Function

In previous sections, we provide groundwork for interpretation of the demand function that is approached with the EoS. Hence, important contribution is to provide the opportunity in applying thermodynamics for describing market equilibrium in economics. In this section, we introduce method of Monte-Carlo simulation of demand function as an example. Procedures in applying of Monte-Carlo techniques are described as follow.

A process to be modeled with Monte-Carlo simulation is a non-deterministic process. The simulation is to imitate real phenomena which is stochastic. Input is random numbers given to the process such that it has also stochastic behavior. This means the output of the each simulation can not be prior-predicted. Since the input is random hence the same executions of the simulation do not generate the same outputs. Many outputs coming out of many executing processes are combined to give statistical implication of the considered phenomena. To perform the simulation, information of probability or distribution function that the system would fall into one way of process is needed (Harrison, 2001; Woolfson & Pert, 1999).

6.4.1 Case of Distribution Function

In statistical mechanics, a form of distribution function, e.g. Maxwell-Boltzmann distribution (or others), can be applied to the simulation, knowing microscopic characters of the system. The particles or molecules' distribution function is known because

1) they are identical,
2) we know properties of each particles and
3) we know how they interact to each other and how they response to external force field.

In our demand-side economics system, we do not know such distribution function (i.e. we do not know microscopic mechanics of the consumers) because consumers are not identical. Properties of each consumer are not exactly known therefore the way each of them interacts and responses to external influence are not known. Lacking of knowledge about microscopic mechanics of consumers in market results in lacking of statistical mechanics of the demand-side economic system.

6.4.2 Case of Discrete Probability

Nucleon or particles' scattering processes in physics are stochastic hence Monte-Carlo simulation is plausible for it. The characteristic scattering time, τ_1 and τ_2 from initial state to state 1 and 2 can be averagely measured. These are time before scattering to state 1 or state 2 to occur. The possible outcomes are scattering to state 1 or state 2 or not being scattered at all. In a time step of $\delta t \ll \tau_1,\ \delta t \ll \tau_2$ probabilities of scattering to state 1 and 2 are

$$\mathrm{Prob}_1 = \frac{1}{\tau_1}\delta t, \tag{6.29}$$

$$\mathrm{Prob}_2 = \frac{1}{\tau_2}\delta t. \tag{6.30}$$

Random number r chosen in the range $(0,1)$ satisfies

$0 < r < \mathrm{Prob}_1$ for scattering to state 1
$\mathrm{Prob}_1 < r < (\mathrm{Prob}_1 + \mathrm{Prob}_2)$ for scattering to state 2
$(\mathrm{Prob}_1 + \mathrm{Prob}_2) < r < 1$ for no scattering.

Number of scattered particles to states 1 and 2 can be simulated using shooting of random numbers to these three possibilities.

In scattering process, characteristic scattering time, τ_1 and τ_2 are characters that reflect internal properties of the aggregated system. They tell us how much chance particles would be scattered into state 1 or 2 in a given time step δt. In our demand-side economics system, instead of considering a given time step δt , we consider a given consumed quantity, δQ^d. At each small fraction of consumed quantity, δQ^d, the characteristic quantities are the change in consumed quantities of goods resulting from either change in price ΔQ^d_{Pr} or change in personal wealth ΔQ^d_{φ}. Instead of considering a time step, δt which is to be compared with characteristic time, τ_1 and τ_2, we consider an amount of quantity of goods consumed, δQ^d and we compare it with characteristic quantities:

$$\Delta Q^d_{Pr} \equiv \frac{Q^d_0}{Pr_0} \Delta Pr E^d_{Pr}, \qquad \text{(where } E^d_{Pr} < 0\text{)}, \tag{6.31}$$

$$\Delta Q^d_{\varphi} \equiv \frac{Q^d_0}{\varphi_0} \Delta\varphi E^d_{\varphi}. \tag{6.32}$$

Just as τ_1 and τ_2 are measured and averaged experimentally, ΔQ^d_{Pr} and ΔQ^d_{φ} have to be evaluated from surveying research. For one unit of changing in price ΔPr (in money unit per unit of good) and changing in personal wealth $\Delta\varphi$ (in money unit per unit of consumer), if data for Q^d_0, φ_0 and Pr_0 can be collected and evaluated, therefore ΔQ^d_{Pr} and ΔQ^d_{φ} can be found.

In a considered range of δQ^d where $\delta Q^d \gg \Delta Q^d_{Pr}$ and $\delta Q^d \gg \Delta Q^d_{\varphi}$ probability of a change in consumed quantity due to the change in price is Prob_1 and probability of a change in consumed quantity due to the change in personal wealth is Prob_2:

$$\text{Prob}_1 = \frac{\Delta Q^d_{Pr}/\delta Q^d}{\left(\Delta Q^d_{Pr}/\delta Q^d\right) + \left(\Delta Q^d_{\varphi}/\delta Q^d\right)}, \tag{6.33}$$

$$\text{Prob}_2 = \frac{\Delta Q^d_{\varphi}/\delta Q^d}{\left(\Delta Q^d_{Pr}/\delta Q^d\right) + \left(\Delta Q^d_{\varphi}/\delta Q^d\right)}, \tag{6.34}$$

where

$$\text{Prob}_1 + \text{Porb}_2 = 1. \tag{6.35}$$

Random number r chosen in the range $(0,1)$ satisfies

$0 < r < \mathrm{Prob}_1$	for change in consumed quantity due to price change
$\mathrm{Prob}_1 < r < 1$	for change in consumed quantity due to change in personal wealth.

Consumed quantity due to change in Pr and φ can hence be simulated by shooting of random numbers to these two range of possibilities. Here changes in consumed quantity are endogenously affected only. Therefore changes in consumed quantity must be from only either changes in price or personal wealth.

The initial values of Pr_0, φ_0, and Q_0^d can be set at the beginning of the simulation. In an ith loop of simulation, $\Delta Q_{\mathrm{Pr,i}}^{\mathrm{d}}, \Delta Pr_i, \Delta Q_{\varphi,i}^d$ and $\Delta\varphi_i$ can be added (updated) accumulately depending on the value of random numbers that falls into the range, $0 < r_i < \mathrm{Prob}_1$ or $\mathrm{Prob}_1 < r_i < 1$.

If the random number falls into the range, $0 < r_i < \mathrm{Prob}_1$, the change ΔPr_i is added to $Pr_i, (i = 0,1, \ldots, n)$ and $\Delta Q_{Pr,i}^d$ is added to $Q_{0,i}^d$. If the random number falls into the range, $\mathrm{Prob}_1 < r_i < 1$, $\Delta\varphi_i$ is added to φ_i and $\Delta Q_{\varphi,i}^d$ is added to $Q_{0,i}^d$. After large n simulations, when equilibrium is reached, the equilibrium value of Q^d, φ and Pr should give the same result as predicted by the EoS (6.23) and (6.27).

The Monte-Carlo technique could provide application to investigate demand function in many markets, especially when there are problems in finding market equilibrium based on the standard empirical measurements in economics. This is the case when performing regression analysis or analyzing of general equilibrium model are neither easy nor possible.

6.5 Chapter Conclusion

We conclude effect structure of variables in physical laws. We use Carathéodory's axiom in formulation of the EoS of the economics system. We consider the effect structure in EoS and propose formal rules to standardize the status of an EoS. We classify the EoS into types of diagram. EoS for a group of consumers in perfect-competitive market is proposed and is tested with the criteria rules proposed here. The new economic EoS passes the criteria and is considered as Class III as it comes with new effect structure diagram. Total demand-side economy's wealth is considered as internal energy. We study the EoS and construct the function $f(Y,T)$ employing econometrics methods. At last we propose a method to perform Monte-Carlo simulation for the EoS.

CHAPTER 7

CONCLUSIONS

There are two main contributions in this thesis. One is on thermodynamics and the other is on thermodynamical formulation of neoclassical economics. In this thesis, we have found new rules to endorse a status of EoS to an empirical equation. This comes with the *effect structure diagram* of the EoS. In invention of the rule, we need to define *truly endogenous function* so that we have solid tool in defining the framework. We apply these new rules and apply the concept of effect structure diagram of the EoS to our proposed EoS. We show that we can use this new concept to disprove the EoS or to advocate the EoS. With the effect structure diagram, we classify the EoS into two classes in thermodynamics. Approaching economics in the style of Carathéodory's axiom hints us to find the EoS and all coordinates prior to considering analogy of optimal quantities as the beginning step. This is because the optimal quantities in both disciplines might stand on or might be defined with different theoretical grounds. The EoS for demand-side economy is found with thermal and mechanical analogous coordinates. These are personal wealth as temperature, price as intensive coordinate and demand quantity as extensive coordinate. We have found that the demand-side EoS is of the new class to be regraded as Class III.

We use this idea to econometrics modeling. Having the EoS helps excluding of non-truly endogenous variables from the model. Therefore modeling demand quantity function must base on the demand-side EoS with only truly endogenous variables in the model, leaving other exogenous effects to be analyzed in the other sets of equations. We propose an econometrics model of the EoS for performing regression analysis. We propose Monte-Carlo simulation method for the EoS and propose the way probability of the simulation is defined. The result of this work is only at beginning step in deeper investigation of economics system with further thermodynamical concepts.

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